

Quarkonium Production in Heavy Ion Collisions: Open Quantum System, Effective Field Theory and Transport Equations

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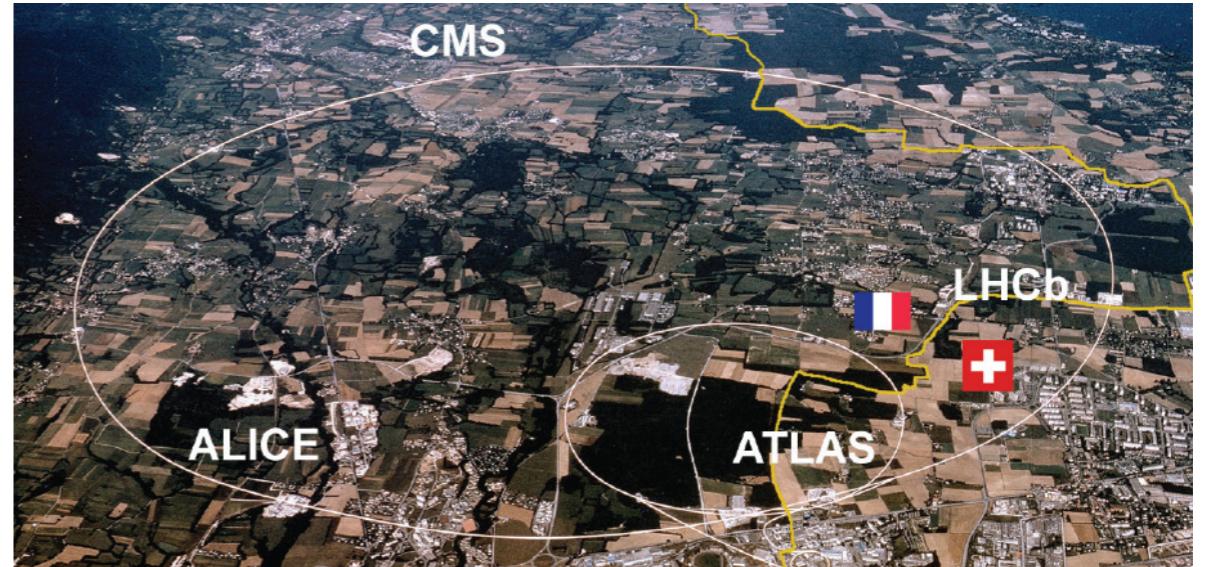
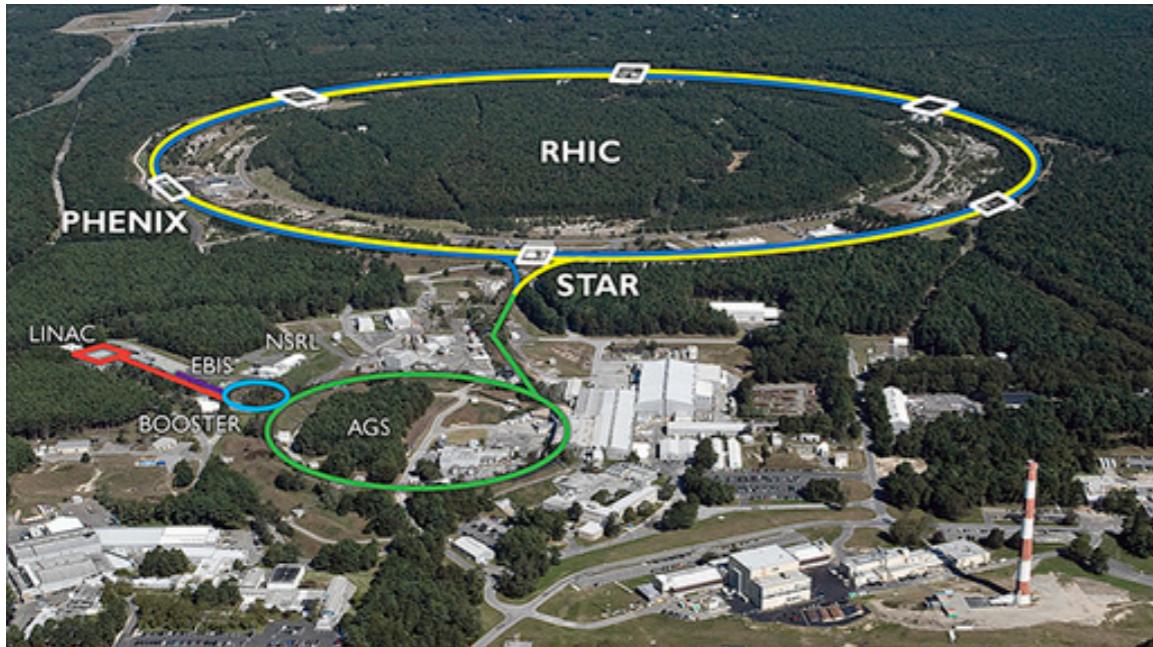
- XY, B.Müller, PRC97 (2018) 1, 014908 (arXiv:1709.03529)
- XY, W.Ke, Y.Xu, S.Bass, B.Müller, Nucl.Phys.A982 (2019) 755 (arXiv:1807.06199)
- XY, T.Mehen, PRD99 (2019) 096028 (arXiv:1811.07027)
- XY, B.Müller, arXiv:1811.09644 (under review, PRD)

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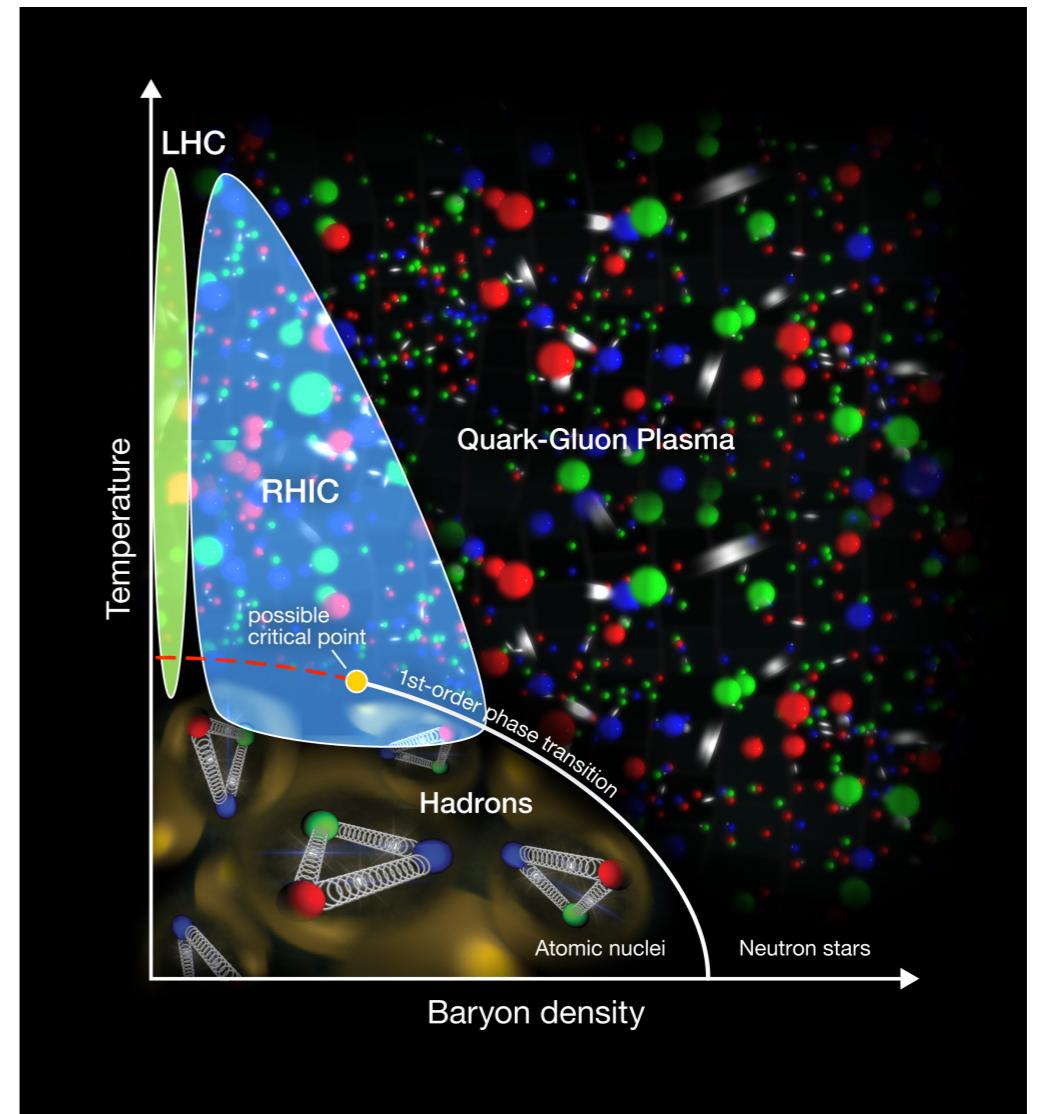
- Introduction: heavy ion collisions, quark-gluon plasma (QGP), quarkonium inside QGP, transport equation
- Derivation of quarkonium transport equation
 - Open quantum system
 - Separation of scales, effective field theory of QCD
- Scattering amplitudes
- Phenomenological studies
- Conclusions

Heavy Ion Collisions and Quark-Gluon Plasma

- Use heavy ion collision experiments to study QGP



- Measure distributions of final-state particles, find QGP signatures
- QGP: deconfined phase of QCD matter, strongly coupled fluid with a small shear viscosity

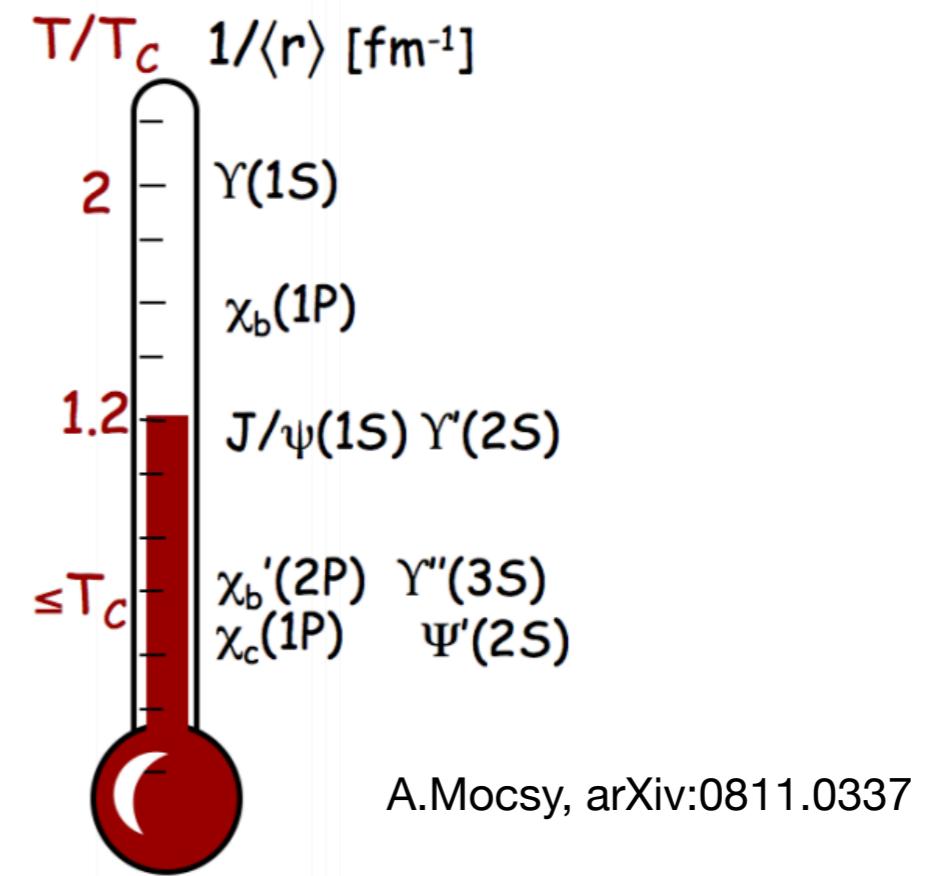
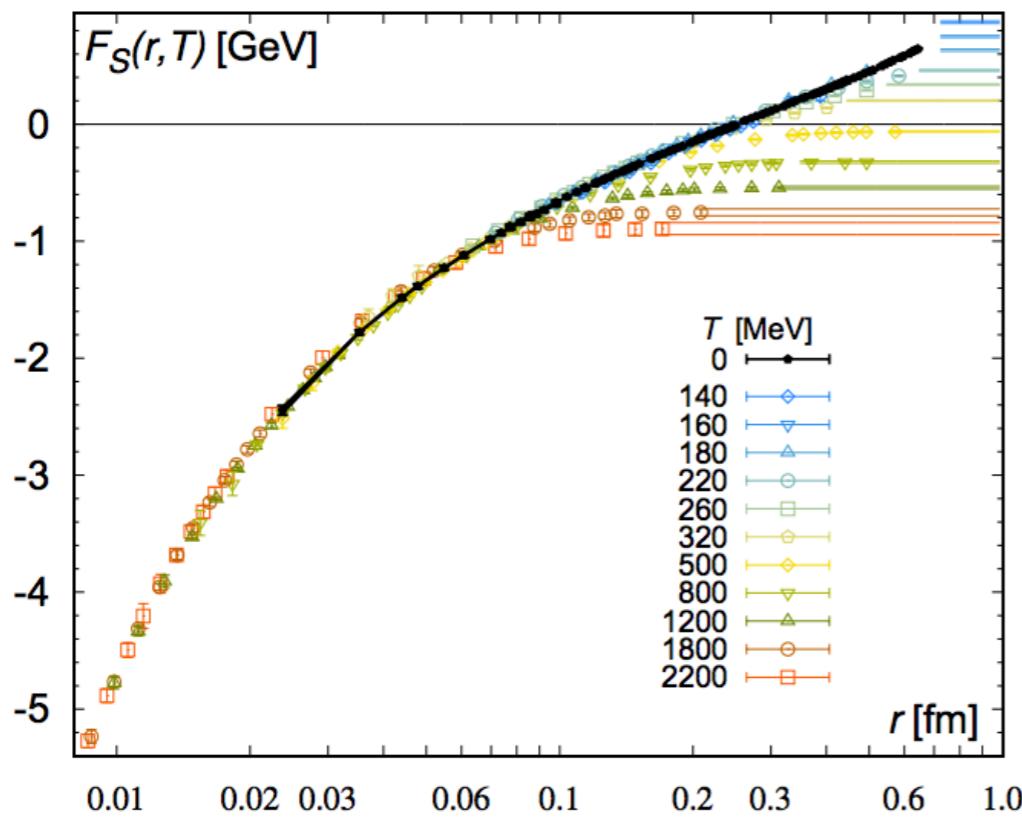


Quarkonium as a Probe of QGP

- Heavy quarkonium: bound state of heavy quark-antiquark pair $Q\bar{Q}$

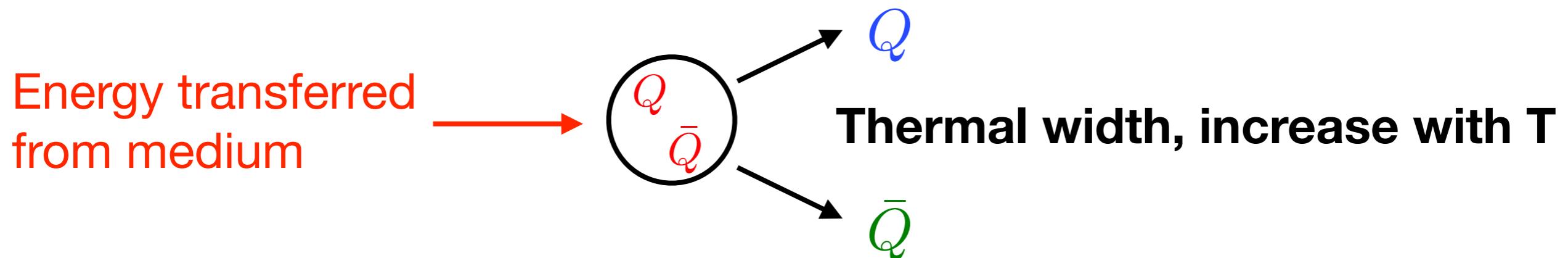
$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 : \text{confining part flattened}$$

- **Static plasma screening**: real part of attractive potential suppressed
—> melting of quarkonium at high temperature —> suppression



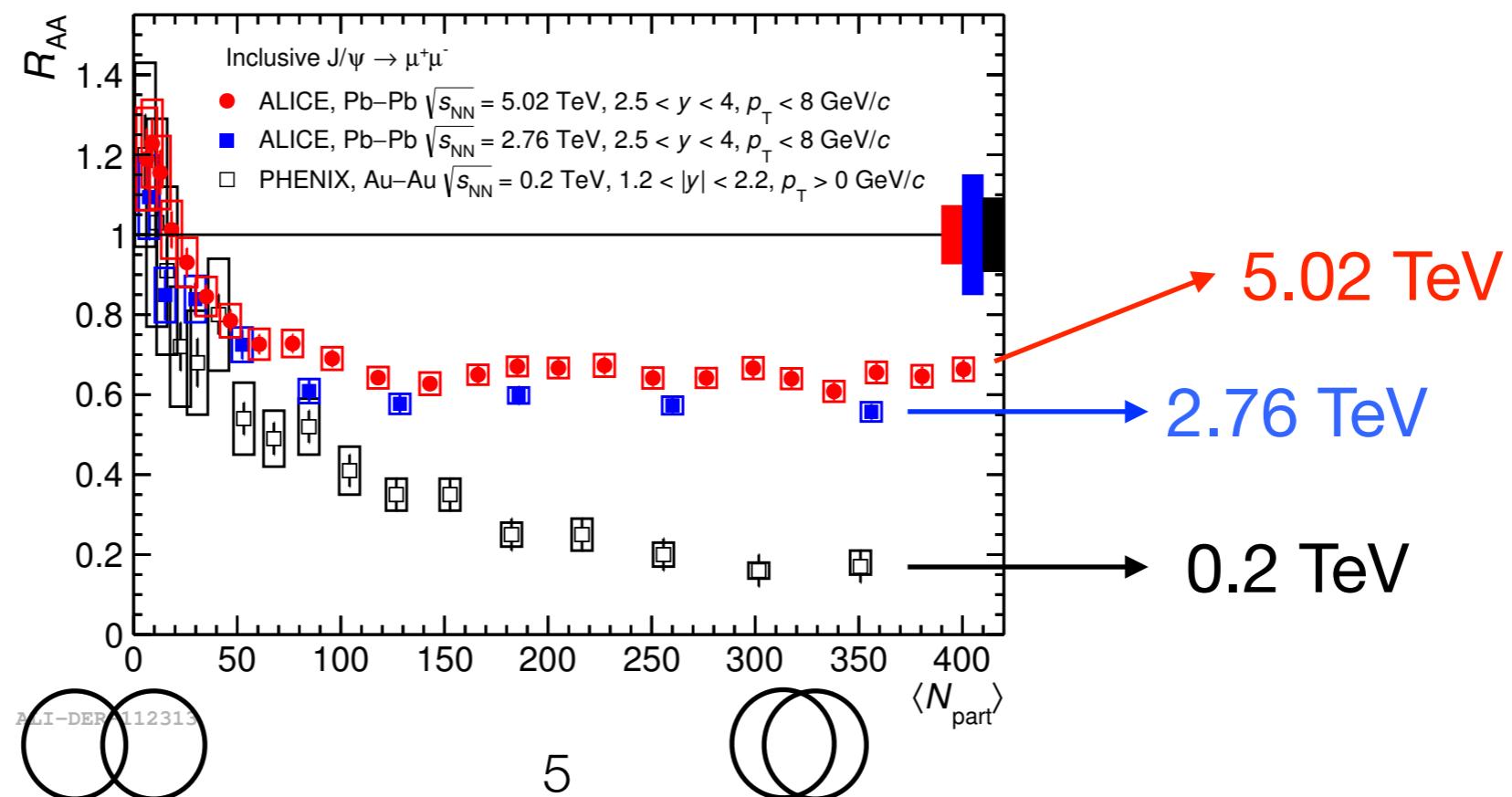
Quarkonium Transport inside QGP

- Dynamical plasma screening: dissociation induced by scattering



- Recombination: inverse process of dissociation, happen inside QGP below melting temperature, J/psi less suppressed at LHC than RHIC

$$R_{AA} \equiv \frac{\sigma_{AA}}{N_{bin} \sigma_{pp}}$$



Quarkonium Transport inside QGP

- Transport equations:

$$\frac{dN}{d\tau} = -\Gamma(T)N + \alpha(\tau)\Gamma(T)N^{\text{eq}}(T)$$

Dissociation rate calculated from QCD

Recombination modeled: detailed balance, phenomenological factor

- Phenomenological success, but need improvement:

Dissociation and recombination not same theoretical framework

Cannot explain approach to equilibrium

Recombination should depend on real-time distributions of open heavy flavors

Relation to the underlying quantum evolution?

Quarkonium Transport inside QGP

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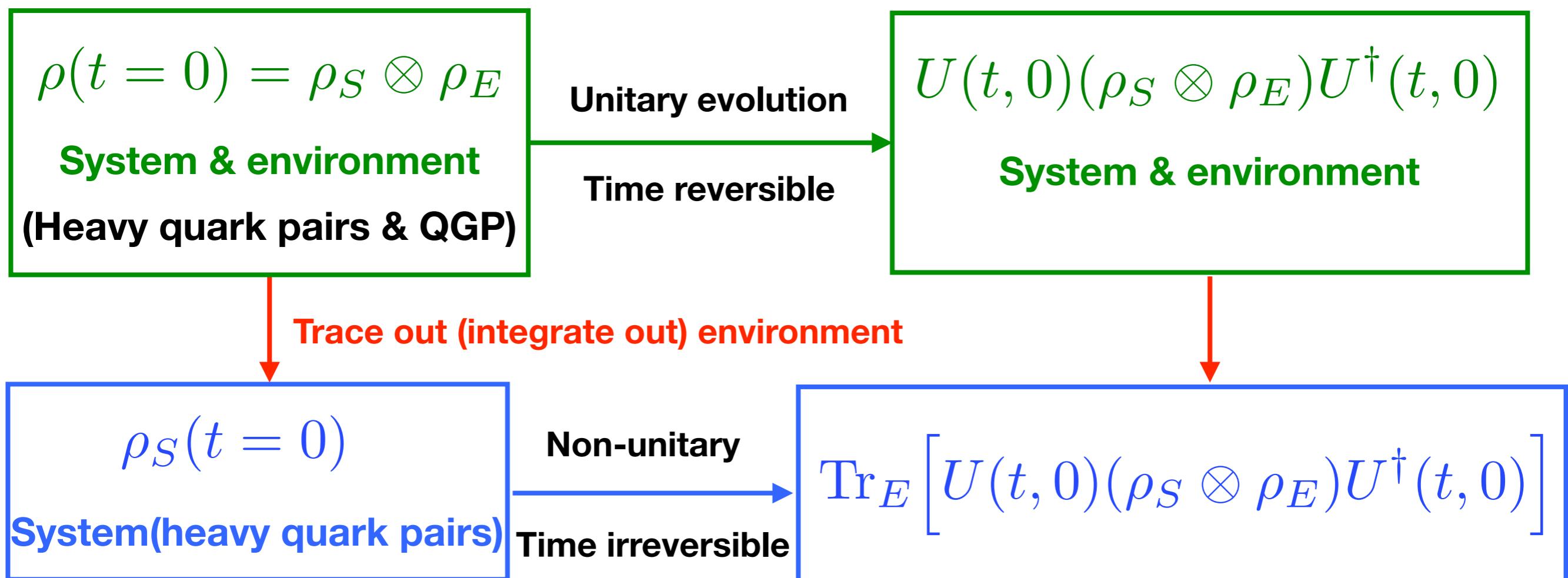
Recombination should depend on real-time distributions of open heavy flavors

address in this talk

Relation to the underlying quantum evolution?

Open Quantum System

$$H = H_S + H_E + H_I$$



$$U(t,0) = \mathcal{T} \exp\left\{-i \int_0^t H_I(t') dt'\right\}$$

From Open Quantum System to Transport Equation

Lindblad equation:

Assume weak coupling $H_I = \sum_{\alpha} O_{\alpha}^{(S)} \otimes O_{\alpha}^{(E)}$

$$\rho_S(t) = \rho_S(0) - i \left[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left(L_{ab} \rho_S(0) L_{cd}^{\dagger} - \frac{1}{2} \{ L_{cd}^{\dagger} L_{ab}, \rho_S(0) \} \right)$$

$$\gamma_{ab,cd}(t) \equiv \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \langle a | O_{\beta}^{(S)}(t_2) | b \rangle \langle c | O_{\alpha}^{(S)}(t_1) | d \rangle^*$$

$$\sigma_{ab}(t) \equiv \frac{-i}{2} \sum_{\alpha,\beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \text{sign}(t_1 - t_2) \langle a | O_{\alpha}^{(S)}(t_1) O_{\beta}^{(S)}(t_2) | b \rangle$$

$$C_{\alpha\beta}(t_1, t_2) \equiv \text{Tr}_E(O_{\alpha}^{(E)}(t_1) O_{\beta}^{(E)}(t_2) \rho_E)$$

$|a\rangle$ Eigenstates of H_S

$$L_{ab} = |a\rangle\langle b|$$



Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(\mathbf{x}, \mathbf{p}, t) + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_{nls}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{nls}^+(\mathbf{x}, \mathbf{p}, t) - \mathcal{C}_{nls}^-(\mathbf{x}, \mathbf{p}, t)$$

From Open Quantum System to Transport Equation

Lindblad equation:

Correction to Hamiltonian

$$\rho_S(t) = \rho_S(0) - i \left[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left(L_{ab} \rho_S(0) L_{cd}^\dagger - \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S(0) \} \right)$$



Markovian approximation

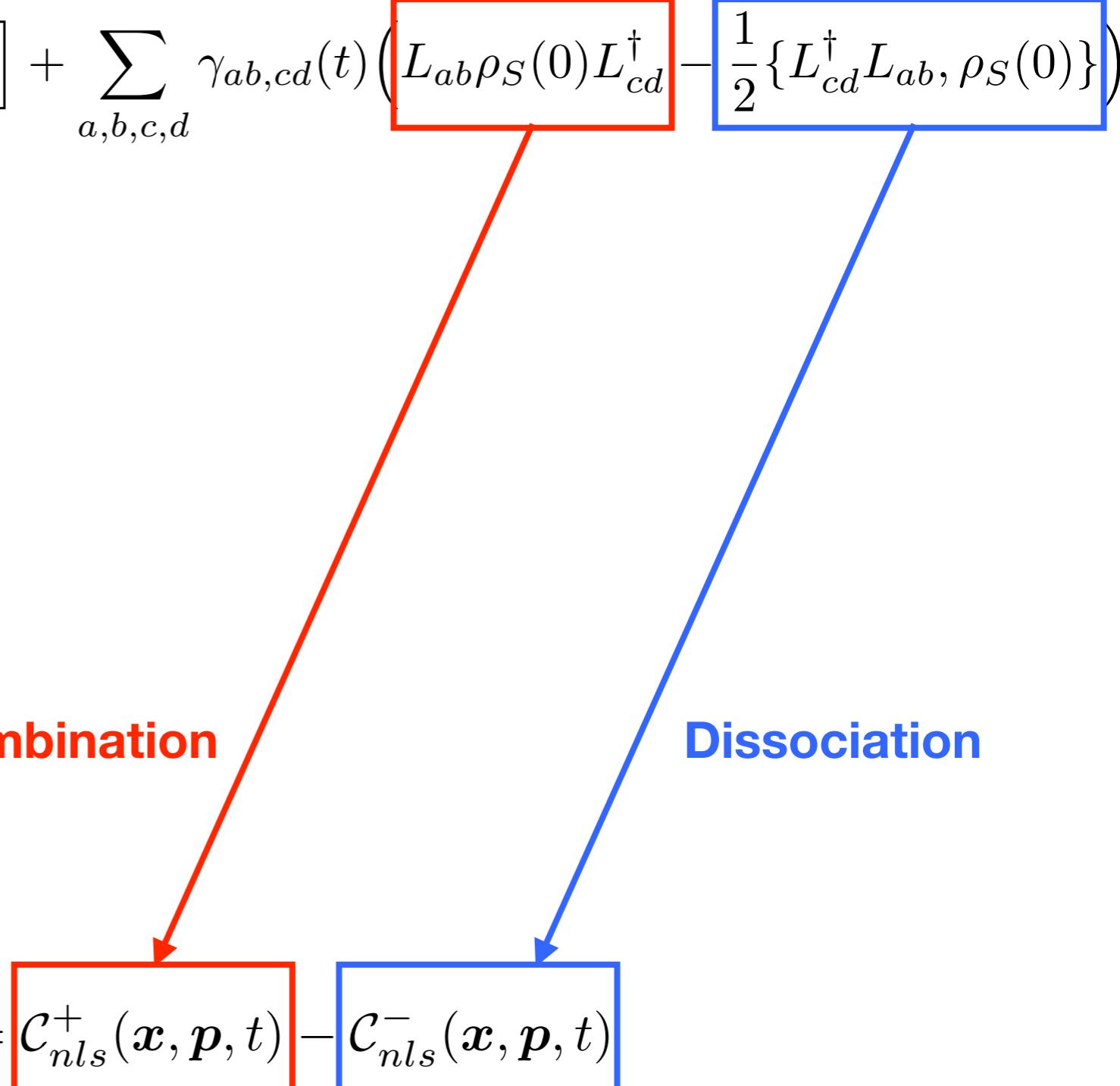
Wigner transform

Boltzmann transport equation

$$\frac{\partial}{\partial t} f_{nls}(x, p, t) + \mathbf{v} \cdot \nabla_x f_{nls}(x, p, t) = \mathcal{C}_{nls}^+(x, p, t) - \mathcal{C}_{nls}^-(x, p, t)$$

Recombination

Dissociation



Two Key Assumptions

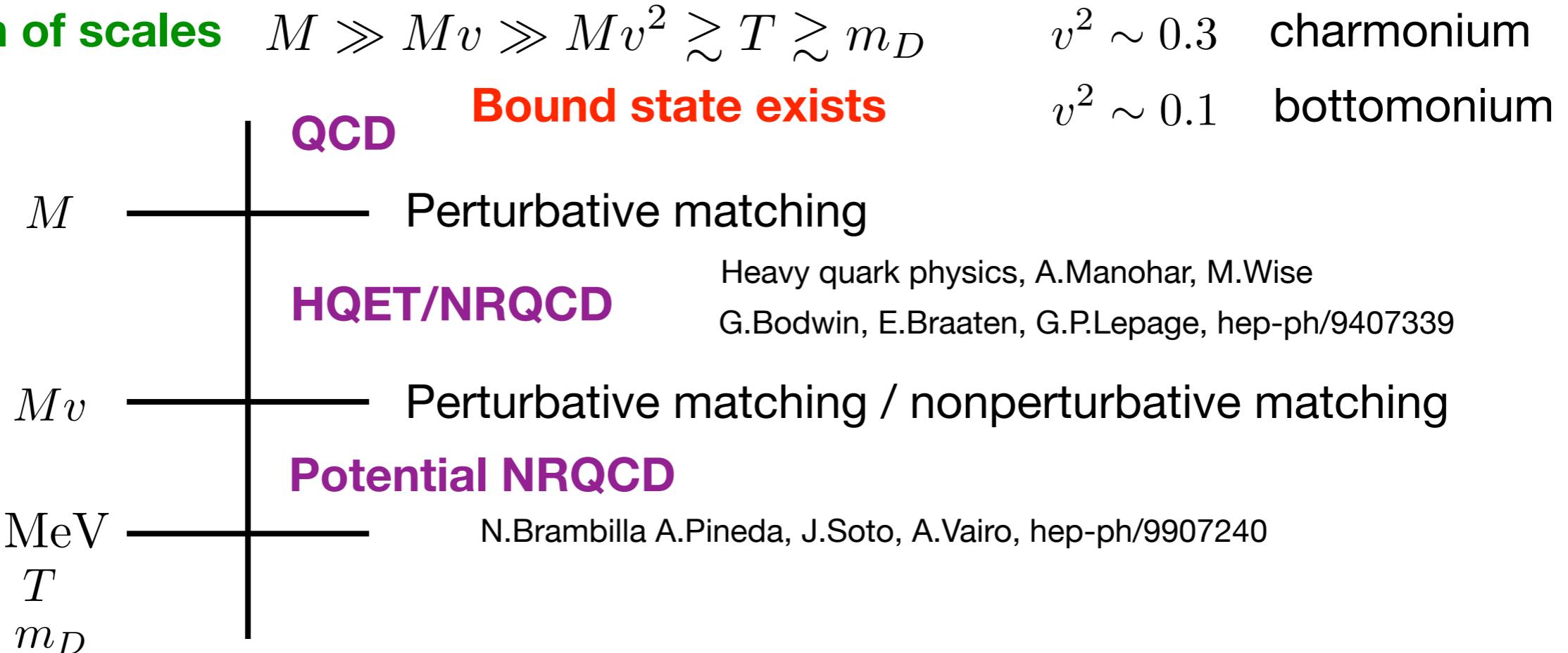
- 1. System interacts weakly with environment ?**

- 2. Markovian assumption (no memory effect) ?**

Effective field theory and separation of scales

Potential NRQCD

Separation of scales



NR & multipole expansions

up to linear order of r

$$S(\mathbf{R}, \mathbf{r}, t) \quad O(\mathbf{R}, \mathbf{r}, t)$$

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr} \left(S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + V_A (O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \dots \right)$$

$$H_s = \frac{(i\nabla_{\text{cm}})^2}{4M} + \frac{(i\nabla_{\text{rel}})^2}{M} + V_s^{(0)} + \frac{V_s^{(1)}}{M} + \frac{V_s^{(2)}}{M^2} + \dots \xrightarrow{\quad} H_{s,o} = \frac{(i\nabla_{\text{rel}})^2}{M} + V_{s,o}^{(0)}$$

- In quarkonium c.m. frame, c.m. energy suppressed by at least one power of v when $v_{\text{med}} \lesssim \sqrt{1-v}$

- Virial theorem
- No hyperfine splitting

Potential NRQCD

Separation of scales $M \gg Mv \gg Mv^2 \sim T \gtrsim m_D$

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr} \left(S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + \boxed{V_A (O^\dagger \mathbf{r} \cdot g \mathbf{E} S + \text{h.c.})} + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g \mathbf{E}, O \} + \dots \right)$$

Dipole interaction $r \sim \frac{1}{Mv}$

Weak coupling between quarkonium and QGP: quarkonium small in size

$$rMv^2 \sim rT \sim rm_D \sim v \text{ suppressed}$$

Potential NRQCD

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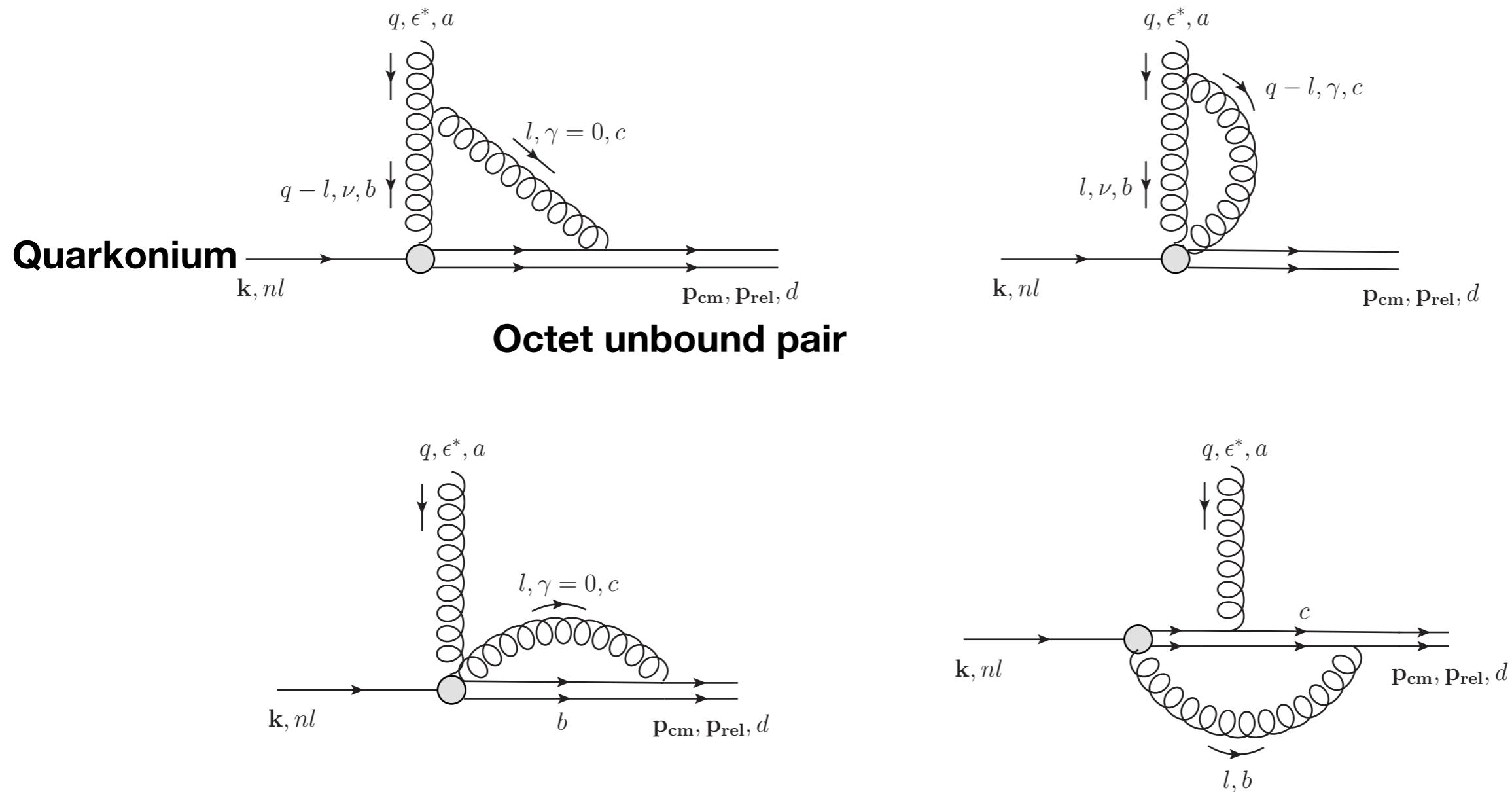
$$rMv^2 \sim rT \sim rm_D \sim v \text{ suppressed}$$



Perturbative matching gives $V_A(\mu = Mv) = 1$

Running? large log? not at one loop

Running of Dipole Interaction



$$\frac{0}{\epsilon} + \dots \quad \frac{d}{d\mu} V_A(\mu) = 0$$

A. Pineda and J. Soto, Phys. Lett. B 495, 323 (2000)

Derivation of Boltzmann Transport Equation

- Correspondence between pNRQCD operators and the general Lindblad equation
- Wigner transform on system density matrix —> phase space distribution
- Markovian approximation, justification from separation of scales
- Write down free streaming and collision terms of transport equation
- Check gauge dependence and infrared safety of scattering amplitudes contributing to the collision terms

Mapping Operators

In general theory

$$H_I = \sum_{\alpha} O_{\alpha}^{(S)} \otimes O_{\alpha}^{(E)} \quad L_{ab} \equiv |a\rangle\langle b|$$

$$\rho_S(t) = \rho_S(0) - i \left[H_S + \sum_{ab} \sigma_{ab}(t) L_{ab}, \rho_S(0) \right] + \sum_{a,b,c,d} \gamma_{ab,cd}(t) \left(L_{ab} \rho_S(0) L_{cd}^{\dagger} - \frac{1}{2} \{ L_{cd}^{\dagger} L_{ab}, \rho_S(0) \} \right)$$

In pNRQCD

$$O_{\alpha}^{(S)} \rightarrow \langle S(\mathbf{R}, t) | r_i | O^a(\mathbf{R}, t) \rangle + \langle O^a(\mathbf{R}, t) | r_i | S(\mathbf{R}, t) \rangle$$

$$O_{\alpha}^{(E)} \rightarrow \sqrt{\frac{T_F}{N_C}} g E_i^a(\mathbf{R}, t) \quad \sum_{\alpha} \rightarrow \int d^3 R \sum_i \sum_a$$

Complete set of states $|a\rangle$

$$|\mathbf{k}, nl, 1\rangle = a_{nl}^{\dagger}(\mathbf{k}) |0\rangle$$

$$|\mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, 1\rangle = b_{\mathbf{p}_{\text{rel}}}^{\dagger}(\mathbf{p}_{\text{cm}}) |0\rangle$$

$$|\mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a\rangle = c_{\mathbf{p}_{\text{rel}}}^{a\dagger}(\mathbf{p}_{\text{cm}}) |0\rangle$$

Bound singlet

Unbound singlet

Unbound octet

Wigner transform \rightarrow formulation in phase space, focus on bound state

$$f_{nl}(\mathbf{x}, \mathbf{k}, t) \equiv \int \frac{d^3 k'}{(2\pi)^3} e^{i\mathbf{k}' \cdot \mathbf{x}} \langle \mathbf{k} + \frac{\mathbf{k}'}{2}, nl, 1 | \rho_S(t) | \mathbf{k} - \frac{\mathbf{k}'}{2}, nl, 1 \rangle$$

Need to calculate

$$\langle \mathbf{k}_1, n_1 l_1, 1 | \rho_S(t) | \mathbf{k}_2, n_2 l_2, 1 \rangle$$

Dissociation

$$-\gamma_{ab,cd} \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S(0) \}$$

For 1st term: $|d\rangle = |\mathbf{k}_1, n_1 l_1, 1\rangle$ $|a\rangle = |c\rangle = |\mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1\rangle$ $|b\rangle = |\mathbf{k}_3, n_3 l_3, 1\rangle$

Linear order in r : transition between bound singlet & unbound octet

$$\begin{aligned} \gamma_{ab,cd} = & \int d^3 R_1 \int d^3 R_2 \sum_{i_1, i_2, b_1, b_2} \int_0^t dt_1 \int_0^t dt_2 C_{\mathbf{R}_1 i_1 b_1, \mathbf{R}_2 i_2 b_2}(t_1, t_2) \\ & \langle \mathbf{k}_1, n_1 l_1, 1 | \langle S(\mathbf{R}_1, t_1) | r_{i_1} | O^{b_1}(\mathbf{R}_1, t_1) \rangle | \mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1 \rangle \\ & \langle \mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1 | \langle O^{b_2}(\mathbf{R}_2, t_2) | r_{i_2} | S(\mathbf{R}_2, t_2) \rangle | \mathbf{k}_3, n_3 l_3, 1 \rangle \end{aligned}$$

Dissociation

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$$\langle \mathbf{k}_1, n_1 l_1, 1 | \langle S(\mathbf{R}_1, t_1) | r_{i_1} | O^{b_1}(\mathbf{R}_1, t_1) \rangle | \mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1 \rangle$$

$$\langle \mathbf{p}_{\text{cm}}, \mathbf{p}_{\text{rel}}, a_1 | \langle O^{b_2}(\mathbf{R}_2, t_2) | r_{i_2} | S(\mathbf{R}_2, t_2) \rangle | \mathbf{k}_3, n_3 l_3, 1 \rangle$$



$$\langle \Psi_{\mathbf{p}_{\text{rel}}} | r_{i_2} | \psi_{n_3 l_3} \rangle \delta^{a_1 b_2} e^{-i(E_{\mathbf{k}_3} t_2 - \mathbf{k}_3 \cdot \mathbf{R}_2)} e^{i(E_{\mathbf{p}} t_2 - \mathbf{p}_{\text{cm}} \cdot \mathbf{R}_2)}$$

$$\frac{T_F}{N_C} g^2 \langle E_{i_1}^{b_1}(\mathbf{R}_1, t_1) E_{i_2}^{b_2}(\mathbf{R}_2, t_2) \rangle_T$$

$$= \frac{T_F}{N_C} g^2 \delta^{b_1 b_2} \int \frac{d^4 q}{(2\pi)^4} e^{iq_0(t_1 - t_2) - i\mathbf{q} \cdot (\mathbf{R}_1 - \mathbf{R}_2)} (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) n_B(q_0) (2\pi) \text{sign}(q_0) \delta(q_0^2 - \mathbf{q}^2)$$

Dissociation

$$-\gamma_{ab,cd} \frac{1}{2} \{ L_{cd}^\dagger L_{ab}, \rho_S(0) \}$$

Markovian approximation: $t \rightarrow \infty$ when doing time integral

Valid when environment correlation time << system relaxation time

$$\begin{array}{c} T^{-1} \\ \text{correlation scale } T \end{array}$$

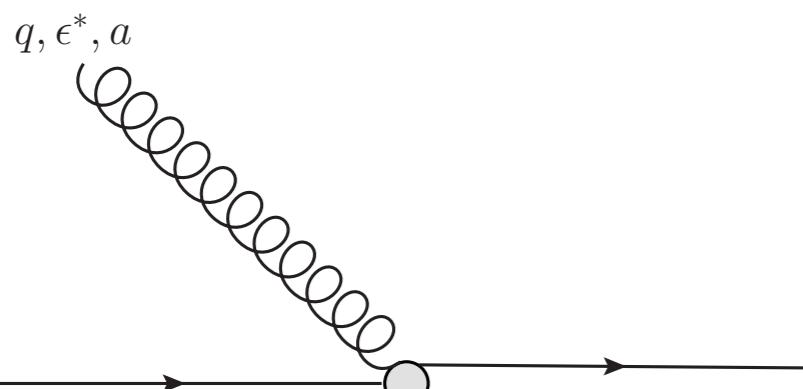
$$\begin{array}{c} \text{dissociation frequency} \\ \text{rate } (grT)^2 T \sim T \frac{\alpha_s T^2}{(Mv)^2} \lesssim \alpha_s v^2 T \end{array}$$

Putting everything together, make Wigner transform:

Spatial & time integrals give delta functions (E&p conservation)

$$\frac{t \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} n_B(q) (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(E_k - E_p + q)}{\frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \mathbf{r} | \Psi_{\mathbf{p}_{\text{rel}}} \rangle|^2 f_{nl}(\mathbf{x}, \mathbf{k}, t=0)}$$

Phase space measure



Amplitude squared

For Coulomb potential, get Peskin-Bhanot result

Recombination

$$\gamma_{ab,cd} L_{ab} \rho_S(0) L_{cd}^\dagger$$

Phase space measure

$$t \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) \sum_{a,i} (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(-|E_{nl}| + q - \frac{\mathbf{p}_{\text{rel}}^2}{M})$$

$$\frac{2T_F}{3N_C} q^2 g^2 \langle \psi_{nl} | r_i | \Psi_{\mathbf{p}_{\text{rel}}} \rangle \int d^3 r \psi_{nl}(\mathbf{r}) r_i \Psi_{\mathbf{p}_{\text{rel}}}^*(\mathbf{r}) f_{Q\bar{Q}}(\mathbf{x}, \mathbf{p}_{\text{cm}}, \mathbf{r}, \mathbf{p}_{\text{rel}}, a, t=0)$$

$$p_{\text{rel}} \sim a_B^{-1} \sim Mv$$

**Amplitude squared, mixed with distribution function
hard to implement numerically**

When can we take distribution function out?

Uniformly distributed when $r <$ Bohr radius a_B $\sqrt{Dt} \gg a_B$

$$D \sim \frac{1}{\alpha_s^2 T} \quad t \sim \frac{a_B}{v_{\text{rel}}} \sim \frac{1}{p_{\text{rel}} v} \quad p_{\text{rel}} \ll \frac{Mv}{\alpha_s^2 v^2}$$

Molecular chaos assumption $f_{Q\bar{Q}}(\mathbf{x}, \mathbf{p}_{\text{cm}}, \mathbf{r}, \mathbf{p}_{\text{rel}}, a, t) = \frac{1}{9} f_Q(\mathbf{x}_1, \mathbf{p}_1, t) f_{\bar{Q}}(\mathbf{x}_2, \mathbf{p}_2, t)$

$$t \frac{1}{9} \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3 2q} (1 + n_B(q)) f_Q(\mathbf{x}_1, \mathbf{p}_1, t) f_{\bar{Q}}(\mathbf{x}_2, \mathbf{p}_2, t)$$

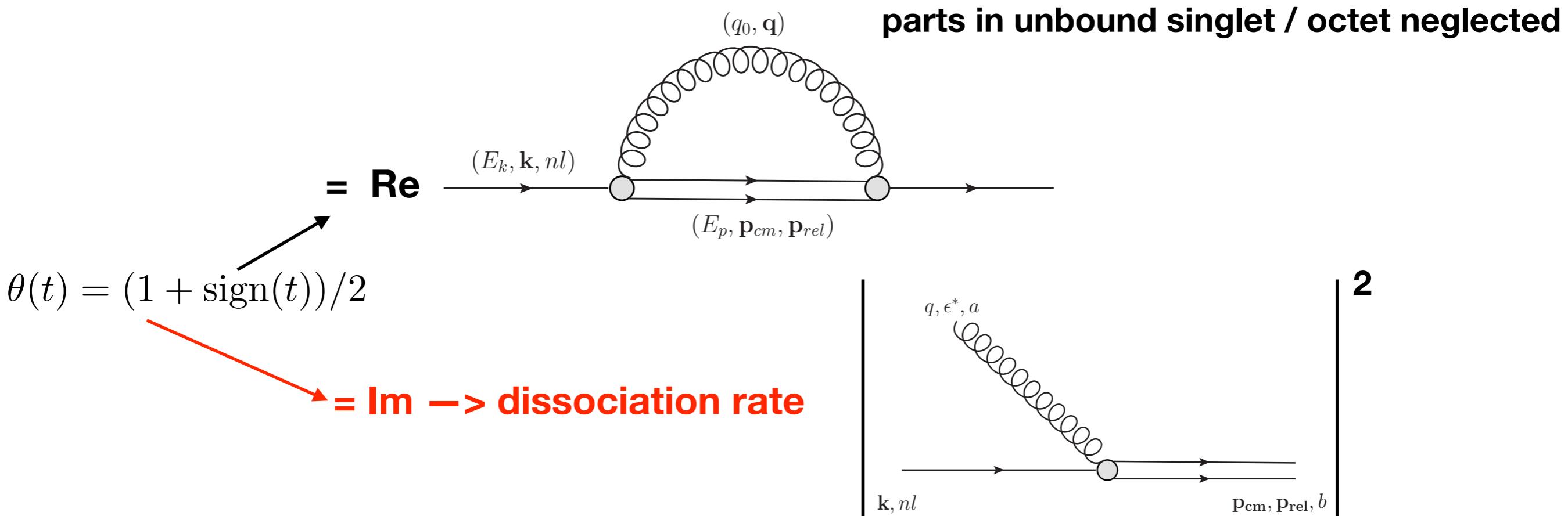
$$(2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} + \mathbf{q}) \delta(-|E_{nl}| + q - \frac{\mathbf{p}_{\text{rel}}^2}{M}) \frac{2}{3} C_F q^2 g^2 |\langle \psi_{nl} | \mathbf{r} | \Psi_{\mathbf{p}_{\text{rel}}} \rangle|^2$$

Spin $g_s = \frac{3}{4}, \frac{1}{4}$

Correction of Potential

$$-i \sum_{ab} \sigma_{ab}(t) [L_{ab}, \rho_S(0)]$$

$$\begin{aligned} \sum_{a,b} \sigma_{ab} L_{ab} &\rightarrow t \sum_{n,l} \int \frac{d^3 k}{(2\pi)^3} \text{Re} \left\{ -ig^2 C_F \sum_{i_1, i_2} \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 p_{\text{cm}}}{(2\pi)^4} \int \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \right. \\ &\quad (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} - \mathbf{q}) \delta(E_k - p_{\text{cm}}^0 - q^0) \\ &\quad (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) \left(\frac{i}{q_0^2 - \mathbf{q}^2 + i\epsilon} + n_B(|q_0|)(2\pi) \delta(q_0^2 - \mathbf{q}^2) \right) \\ &\quad \left. \langle \psi_{nl} | r_{i_1} \frac{i |\Psi_{\mathbf{p}_{\text{rel}}} \rangle \langle \Psi_{\mathbf{p}_{\text{rel}}}|}{p_{\text{cm}}^0 - E_p + i\epsilon} r_{i_2} | \psi_{nl} \rangle \right\} L_{|\mathbf{k}, nl, 1\rangle \langle \mathbf{k}, nl, 1|} \end{aligned}$$



Boltzmann Transport Equation

Phase space free streaming $H_{eff} = H_S + \sum \sigma_{ab} L_{ab}$

$$\rho_S(t) = \rho_S(0) - it(H_{eff}\rho_S(0) \stackrel{a,b}{-} \rho_S(0)H_{eff}) + \dots$$

↓
Wigner transform

$$f_{nl}(x, k, t) = f_{nl}(x, k, 0) - it \int \frac{d^3 k'}{(2\pi)^3} e^{i k' \cdot x} (E_{k+\frac{k'}{2}} - E_{k-\frac{k'}{2}}) \langle k + \frac{k'}{2}, nl, 1 | \rho_S(0) | k - \frac{k'}{2}, nl, 1 \rangle + t \mathcal{C}^+ - t \mathcal{C}^-$$

$$E_{k \pm \frac{k'}{2}} = -|E_{nl}| + \frac{(k \pm \frac{k'}{2})^2}{4M}$$

Add spin dependence → transport equation:

$$\frac{\partial}{\partial t} f_{nls}(x, p, t) + \mathbf{v} \cdot \nabla_x f_{nls}(x, p, t) = \mathcal{C}_{nls}^+(x, p, t) - \mathcal{C}_{nls}^-(x, p, t)$$

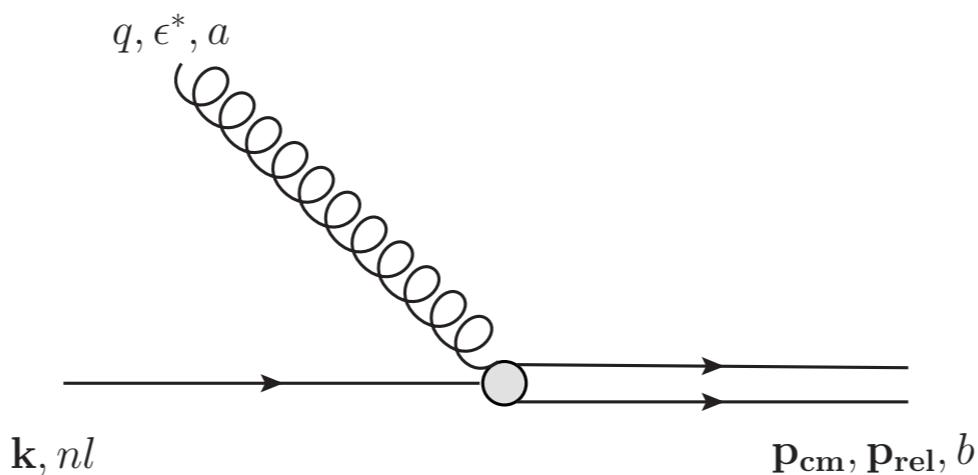
Open quantum system

Effective field theory: separation of scales

Weak coupling between quarkonium & QGP

Markovian approximation

LO Amplitudes



$$i\mathcal{M} = g\sqrt{\frac{T_F}{N_c}}(q^0\epsilon^{*i} - q^i\epsilon^{*0})\langle\Psi_{\mathbf{p}_{\text{rel}}} | r^i | \psi_{nl} \rangle \delta^{ab} \equiv i\epsilon^{*\mu}(\mathcal{M})_\mu$$

Ward identity $q^\mu(\mathcal{M})_\mu = 0$

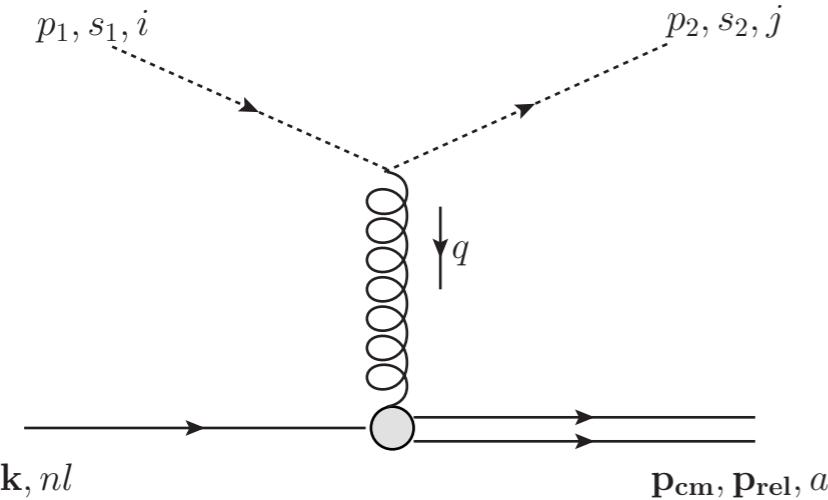
Higher order corrections of

$$\frac{T_F}{N_C}g^2\langle E_{i_1}^{b_1}(\mathbf{R}_1, t_1)E_{i_2}^{b_2}(\mathbf{R}_2, t_2)\rangle_T$$

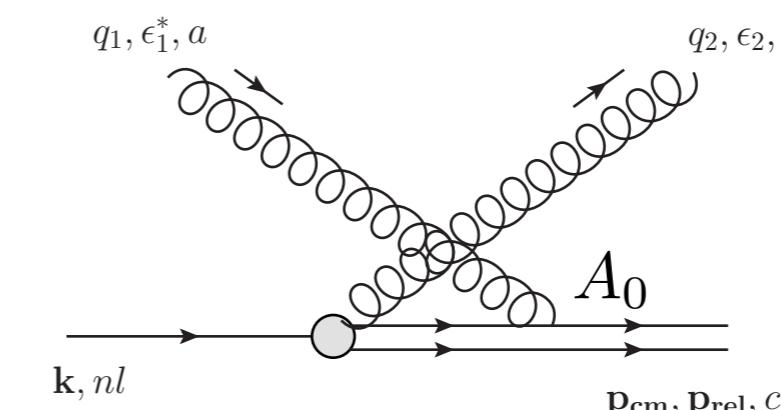
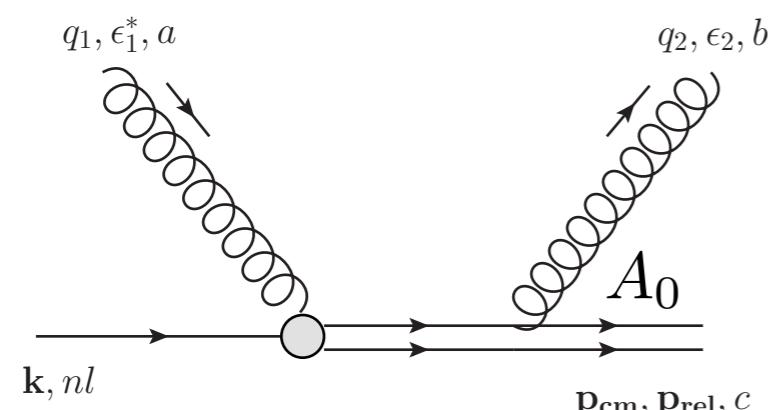
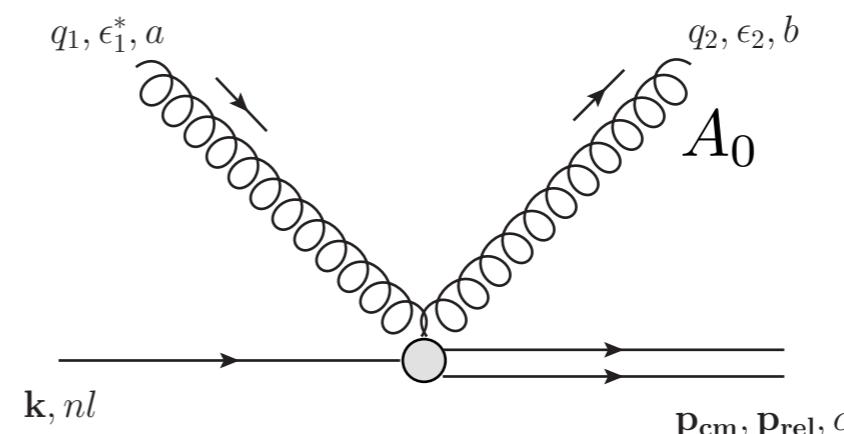
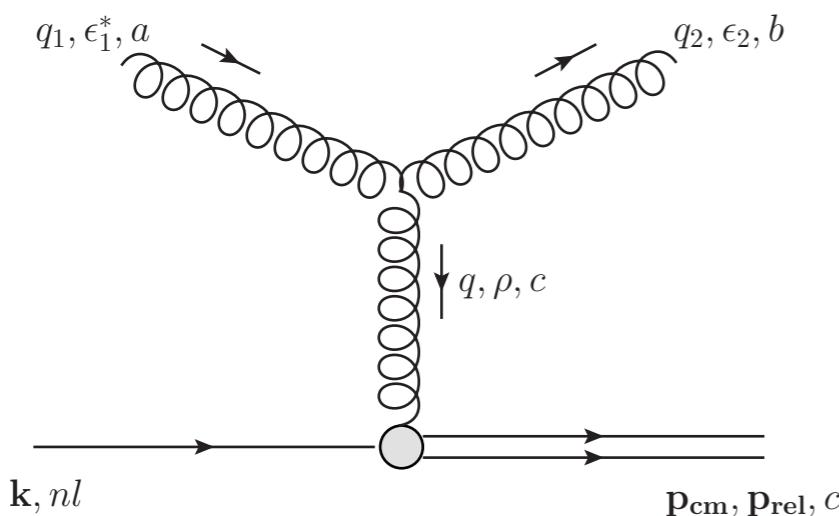


Inelastic scattering in t-channel

NLO Amplitudes: Light Quark and Gluon



**Ward identity
Infrared safety**



Coupled with Transport of Open Heavy Flavor

heavy quark

$$\left(\frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_Q(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

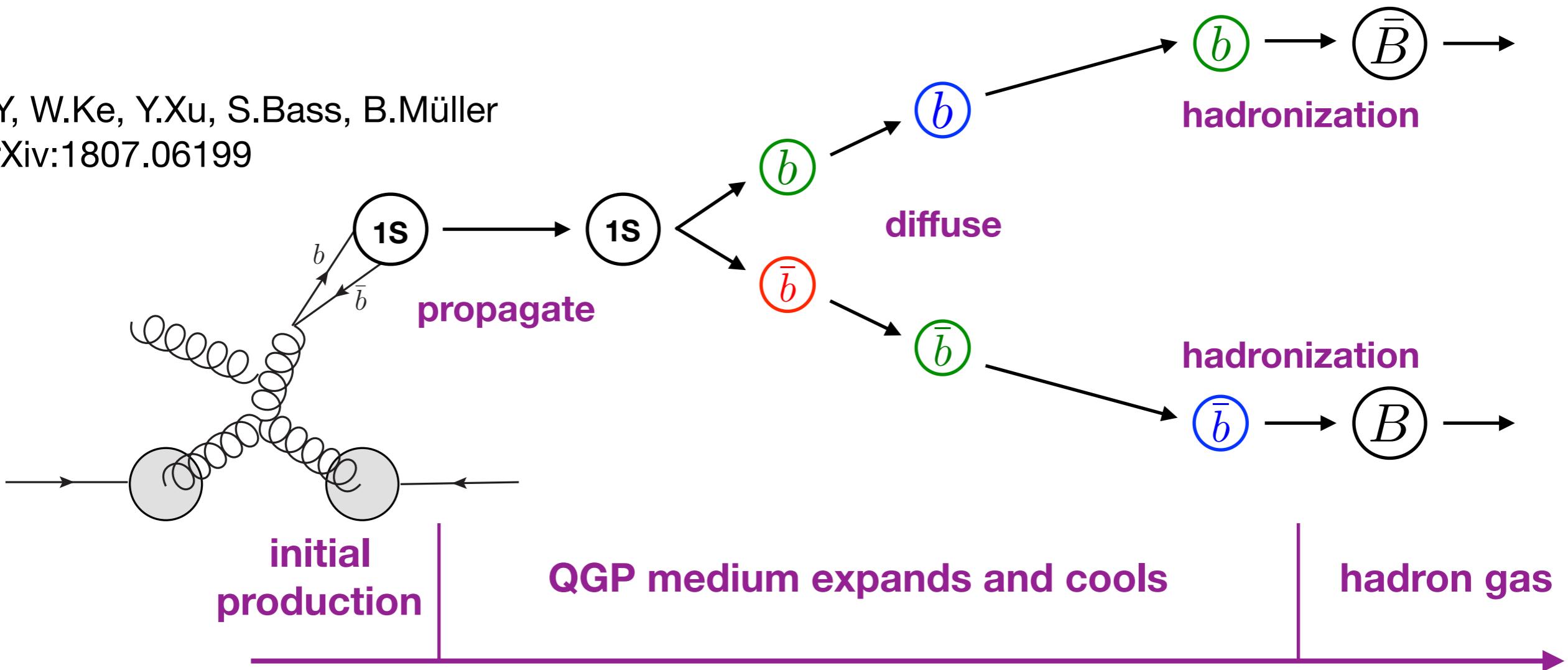
anti-heavy quark

$$\left(\frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_{\bar{Q}}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

each quarkonium state
nl = 1S, 2S, 1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_{nls}(\mathbf{x}, \mathbf{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$

XY, W.Ke, Y.Xu, S.Bass, B.Müller
arXiv:1807.06199



Coupled with Transport of Open Heavy Flavor

heavy quark

$$\left(\frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_Q(x, p, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

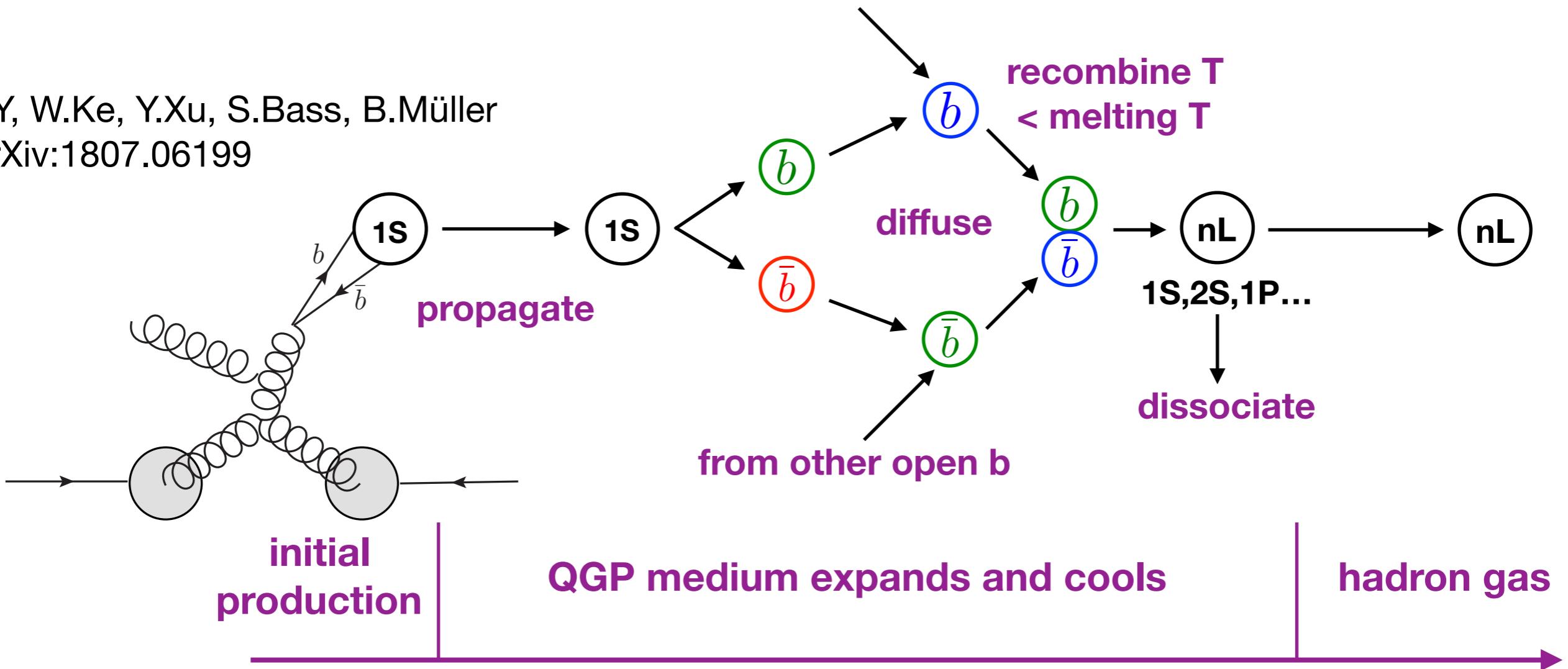
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$$\left(\frac{\partial}{\partial t} + \dot{x} \cdot \nabla_x \right) f_{\bar{Q}}(x, p, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

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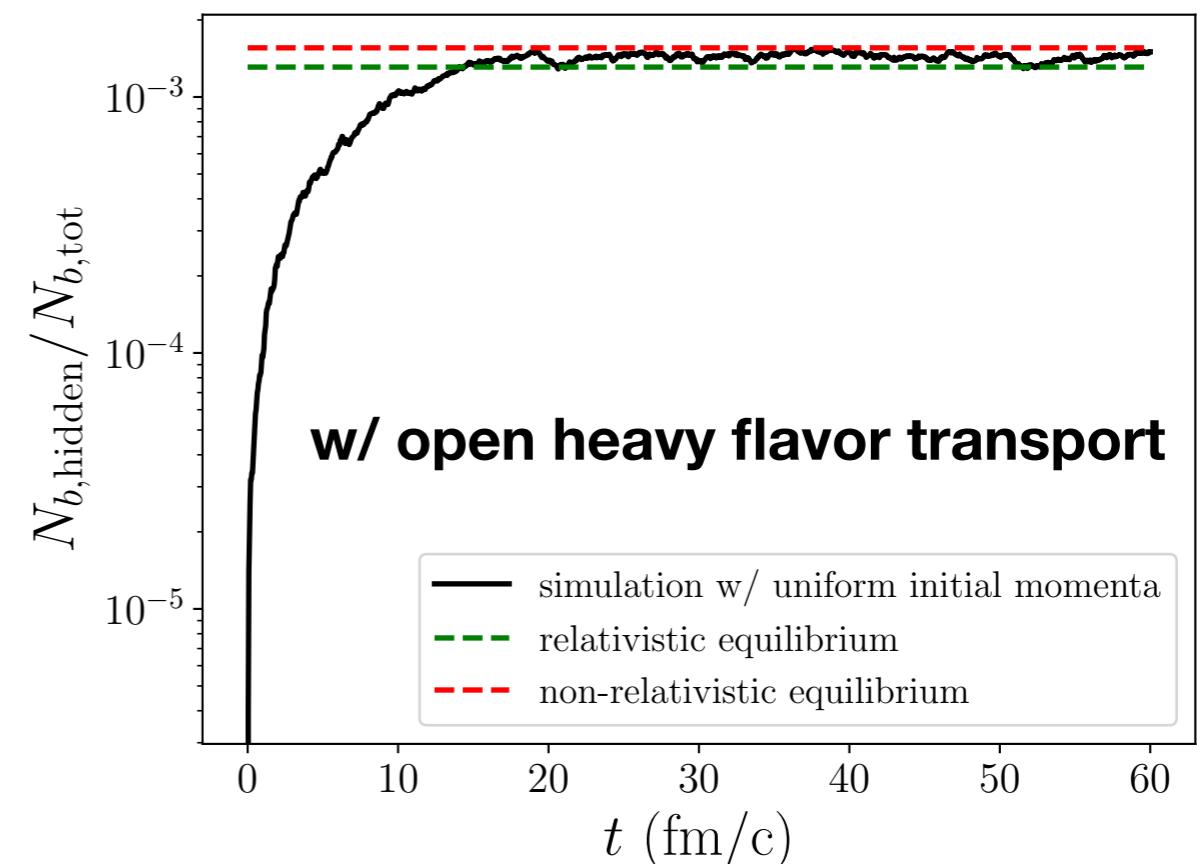
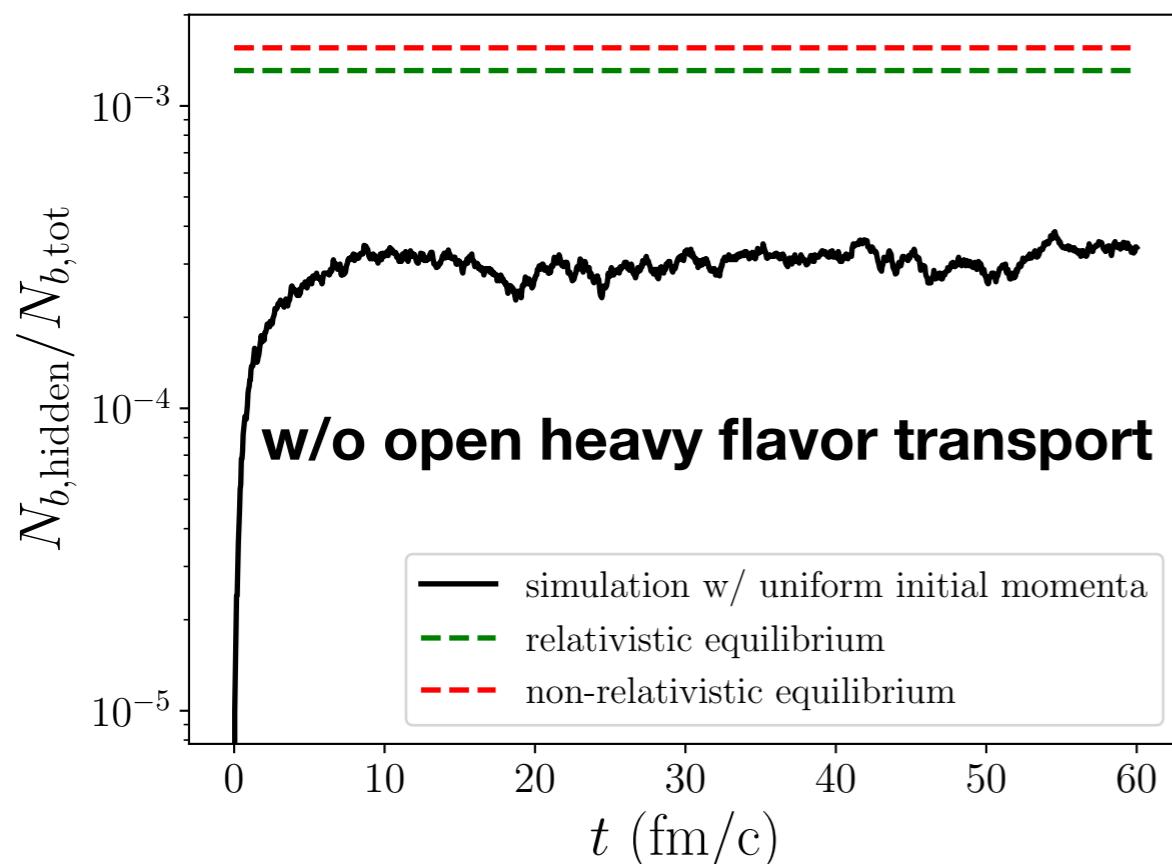
XY, W.Ke, Y.Xu, S.Bass, B.Müller
arXiv:1807.06199



Approach to Equilibrium

Setup:

- QGP box w/ const $T=300$ MeV, 1S state & b quarks, total b flavor = 50 (fixed)
- Initial momenta sampled from uniform distributions 0-5 GeV
- Turn on/off open heavy quark transport



XY, B.Müller, arXiv:1709.03529

Dissociation-recombination
interplay drives to detailed balance

Heavy quark energy gain/loss necessary
to drive kinetic equilibrium of quarkonium

Collision Event Simulation

- Initial production:

PYTHIA 8.2: NRQCD factorization

Sjostrand, et al, Comput. Phys.Commun.191 (2015) 159
Bodwin, Braaten, Lepage Phys. Rev. D 51, 1125 (1995)

Nuclear PDF: EPS09 (cold nuclear matter effect) Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

Trento, sample position, hydro. initial condition

Moreland, Bernhard, Bass, Phys. Rev. C 92, no. 1, 011901 (2015)

- Medium background: 2+1D viscous hydrodynamics (**calibrated**)

Song, Heinz, Phys.Rev.C77,064901(2008)

Shen, Qiu, Song, Bernhard, Bass, Heinz, Comput. Phys. Commun.199,61 (2016)

Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94,no.2,024907(2016)

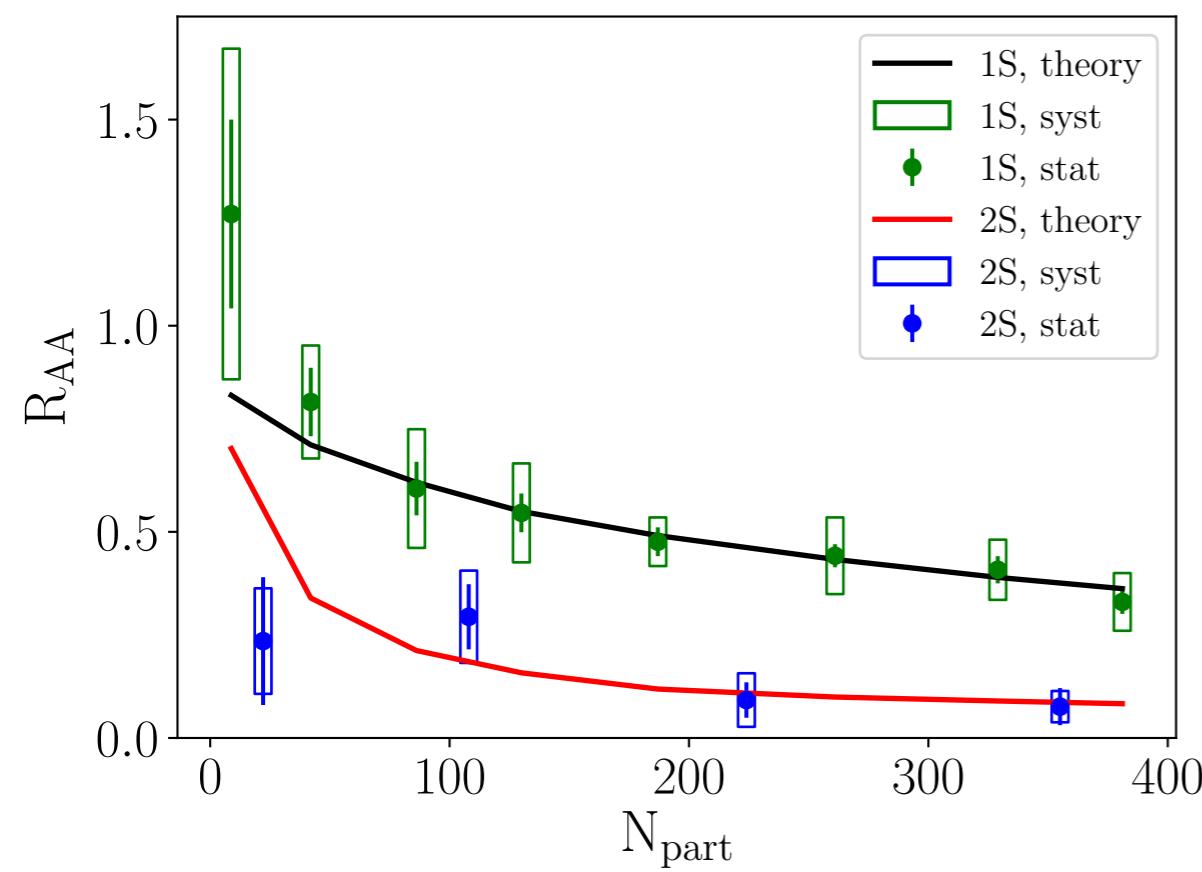
- Study bottomonium (larger separation of scales); include 1S 2S; ~26% 2S feed-down to 1S in hadronic phase (from PDG); initial production ratio 1S : 2S ~ between 3:1 to 4:1 (PYTHIA)

Upsilon in 2760 GeV PbPb Collision

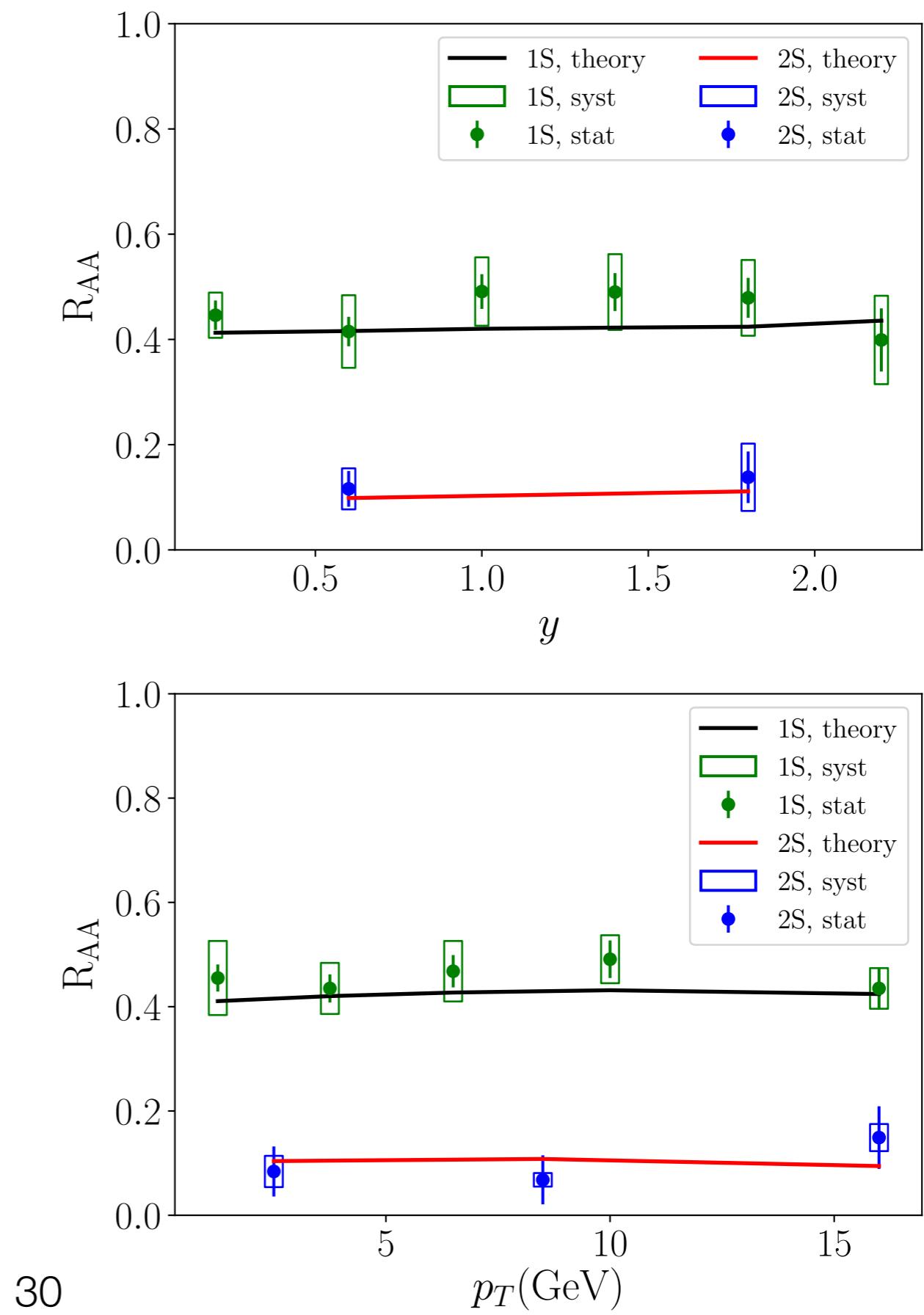
Fix $\alpha_s = 0.3$

Tune $T_{\text{melt}}(2S) = 210 \text{ MeV}$

Tune $V_s = -C_F \frac{0.42}{r}$

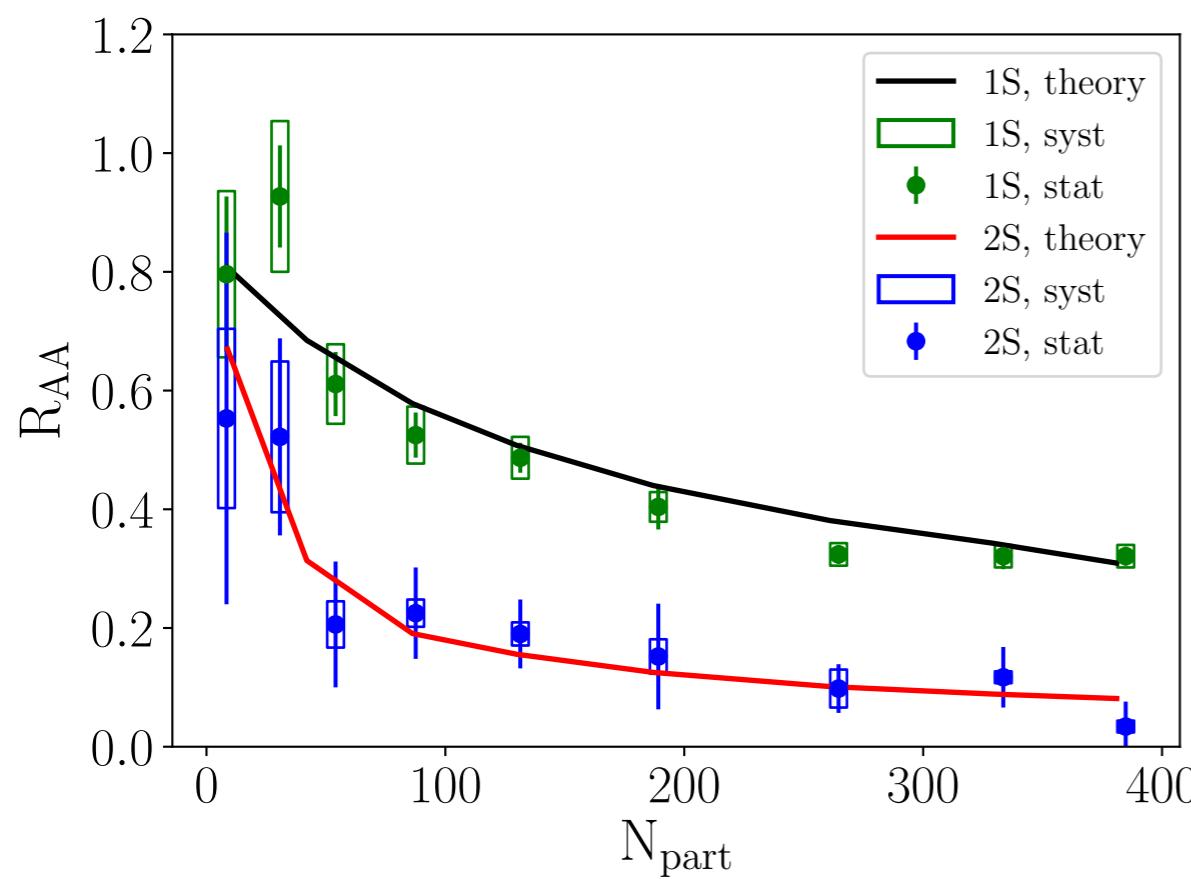


CMS Phys.Lett. B 770 (2017) 357-379

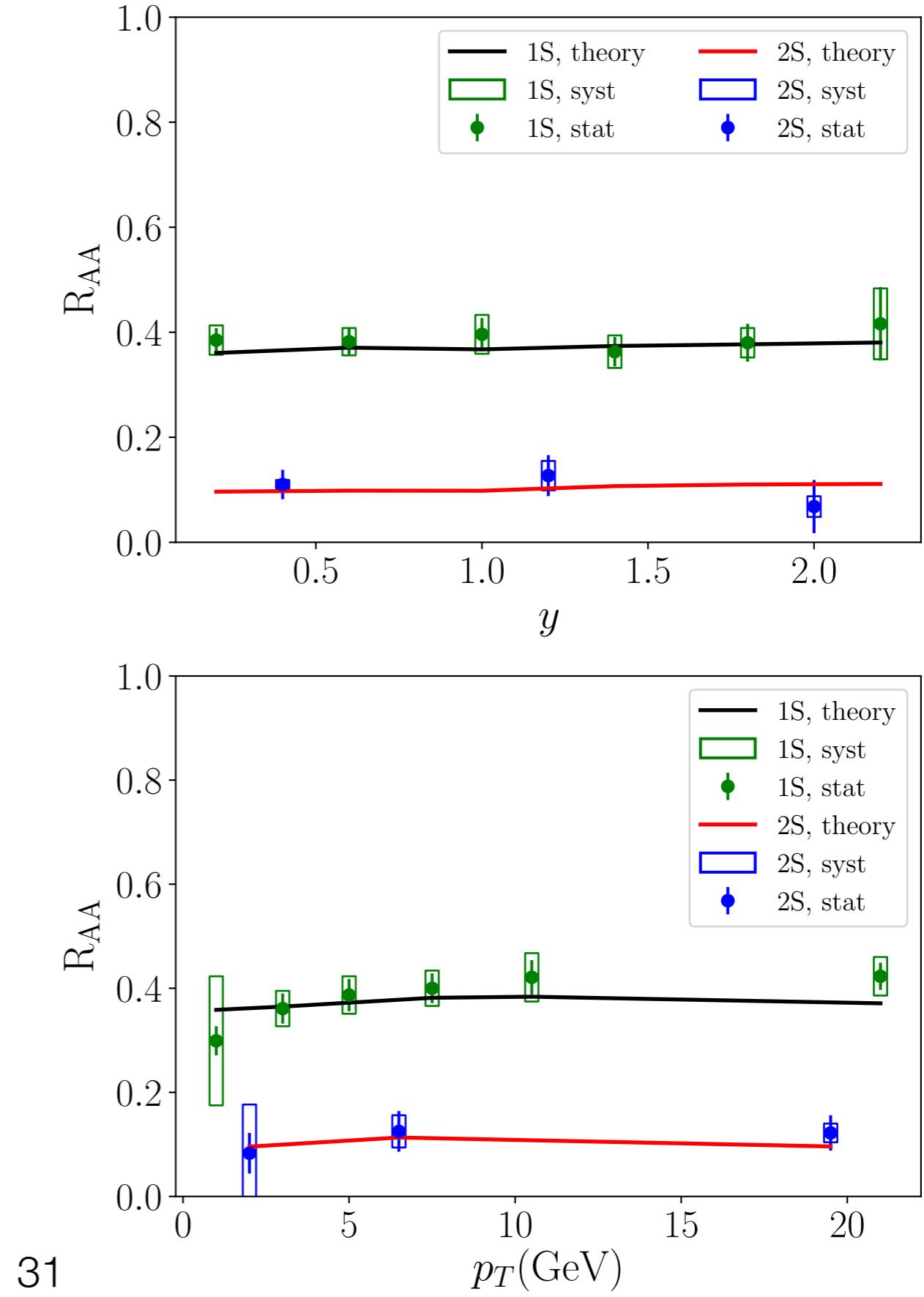


Upsilon in 5020 GeV PbPb Collision

Use same set of parameters

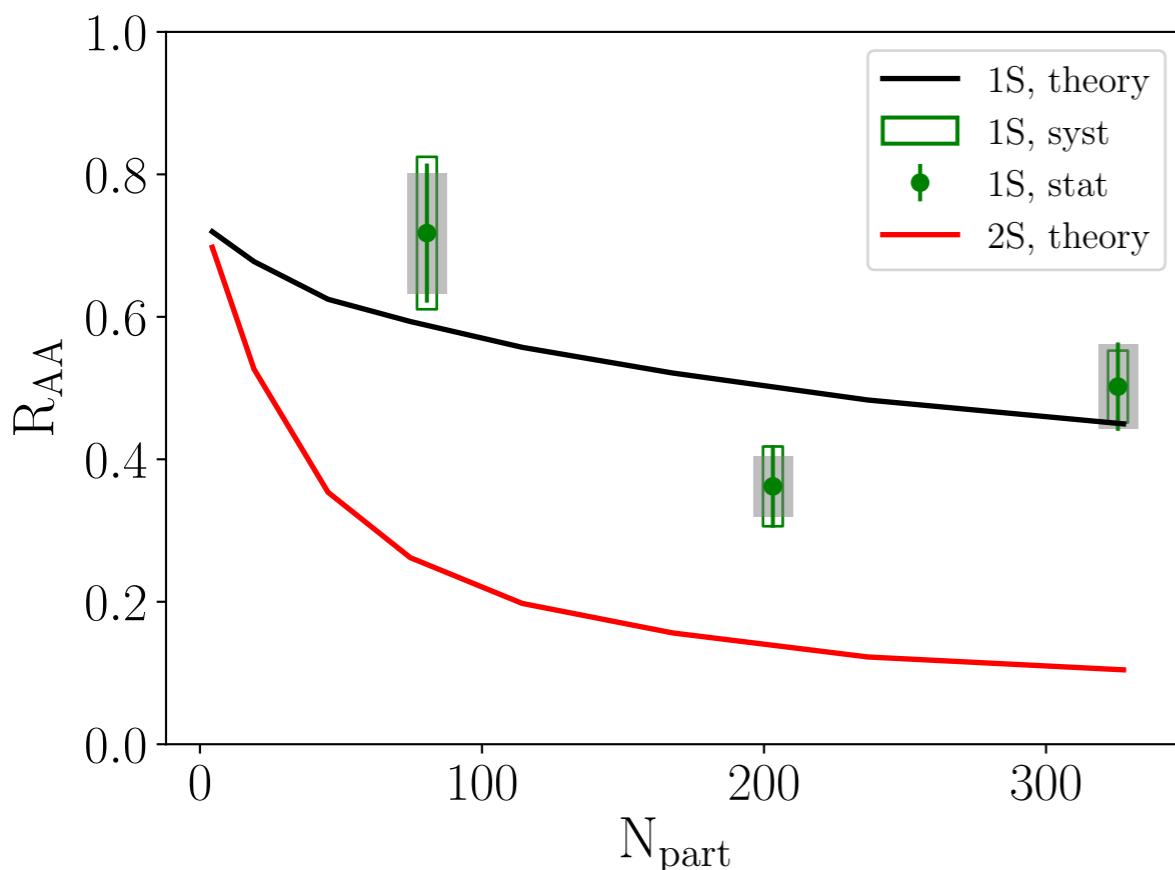


CMS arXiv:1805.09215

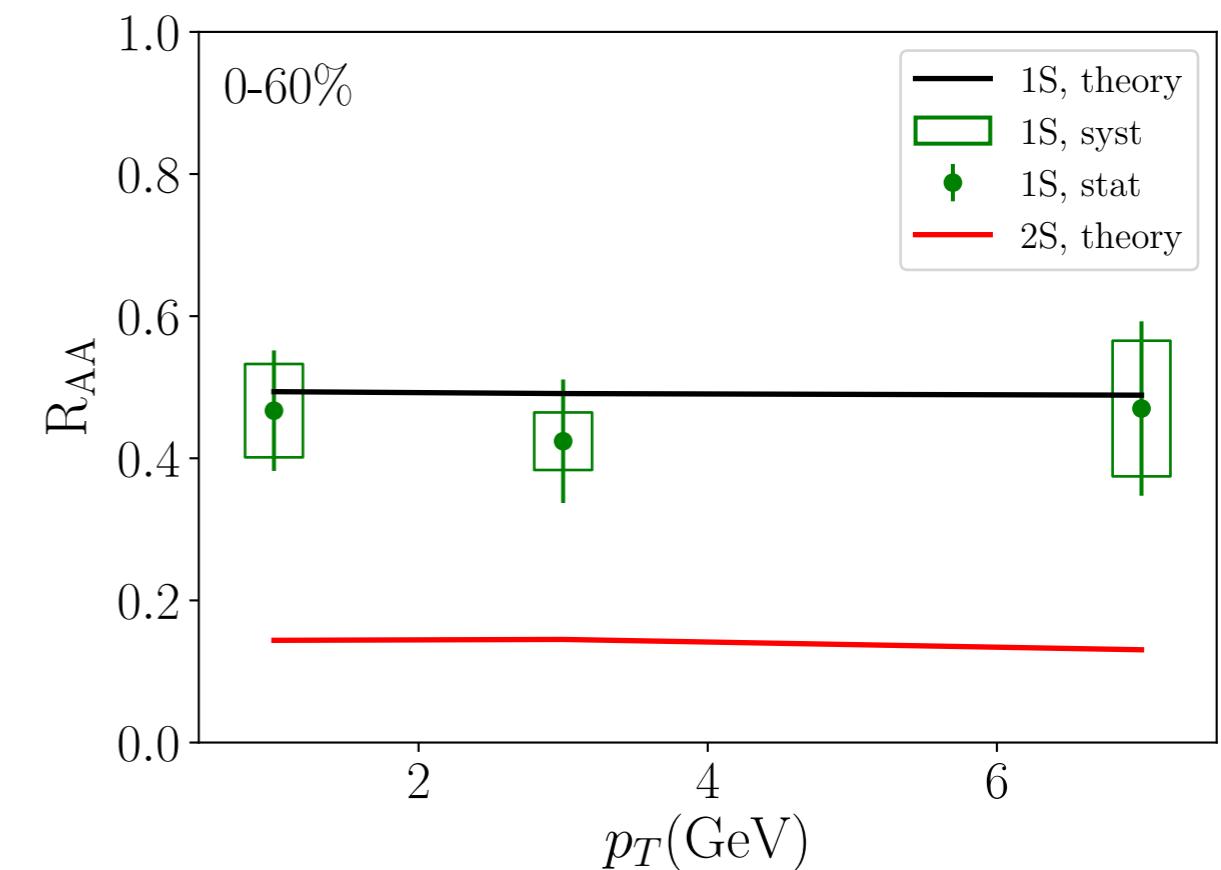


Upsilon in 200 GeV AuAu Collision

Use same set of parameters



STAR measures 2S+3S

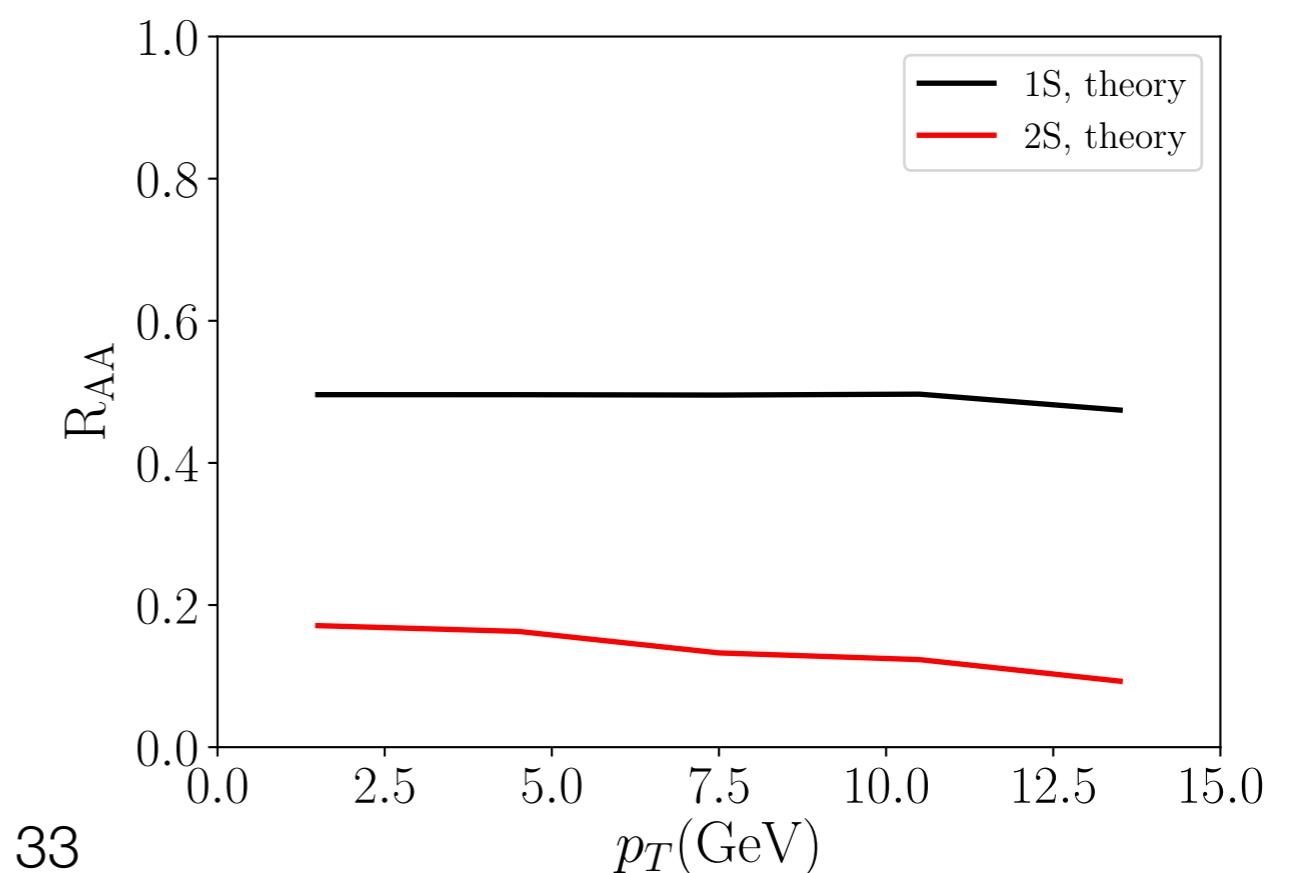
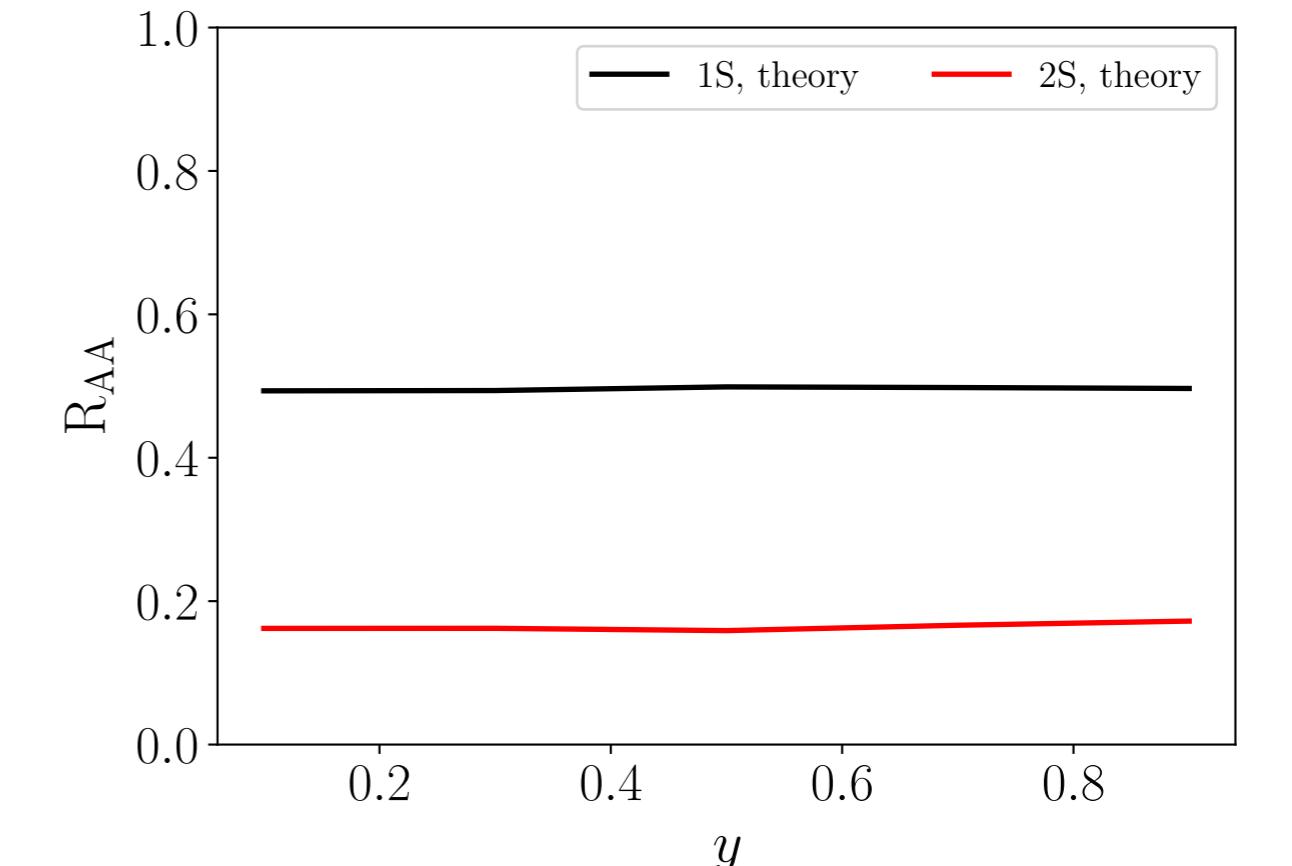
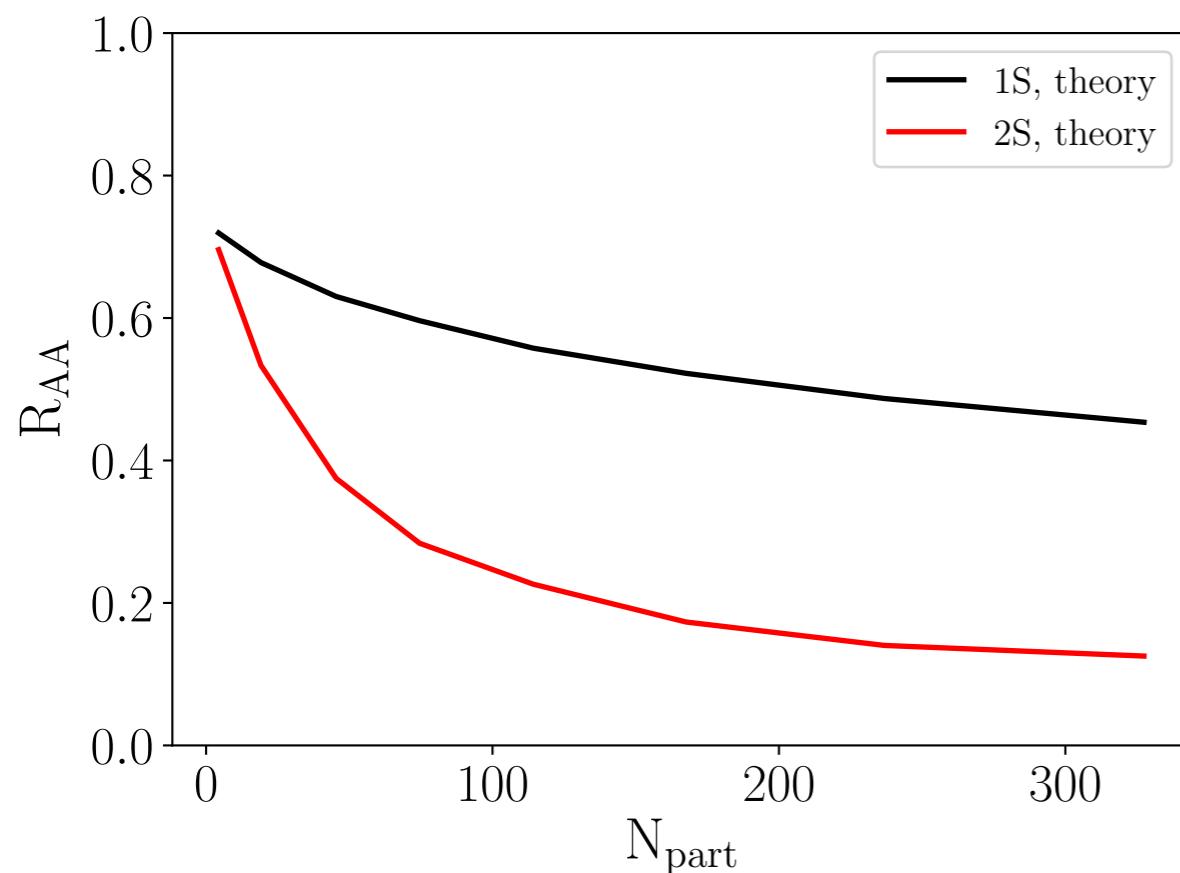


STAR Talks at Quark Matter 17&18

Upsilon in 200 GeV AuAu Collision

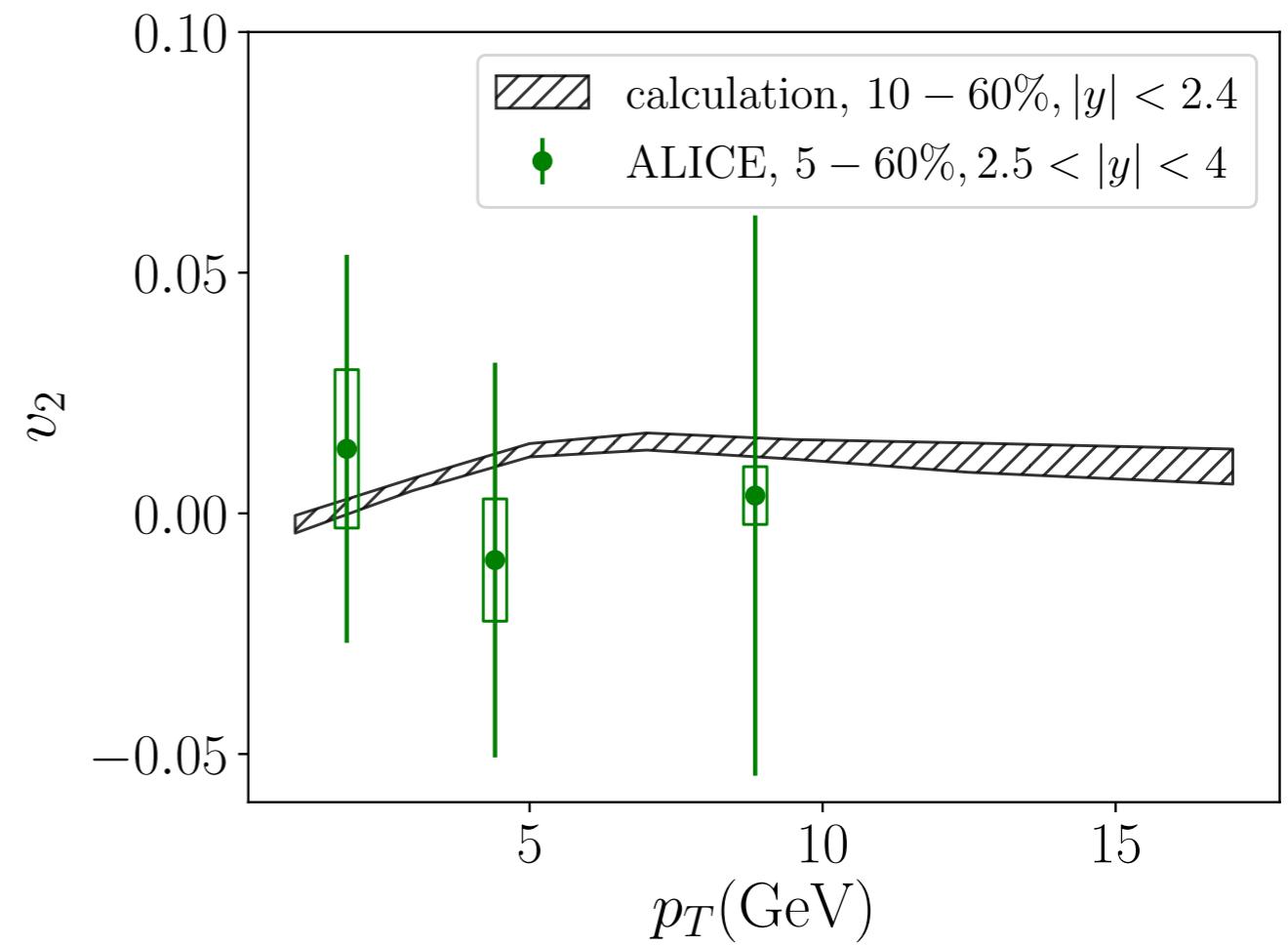
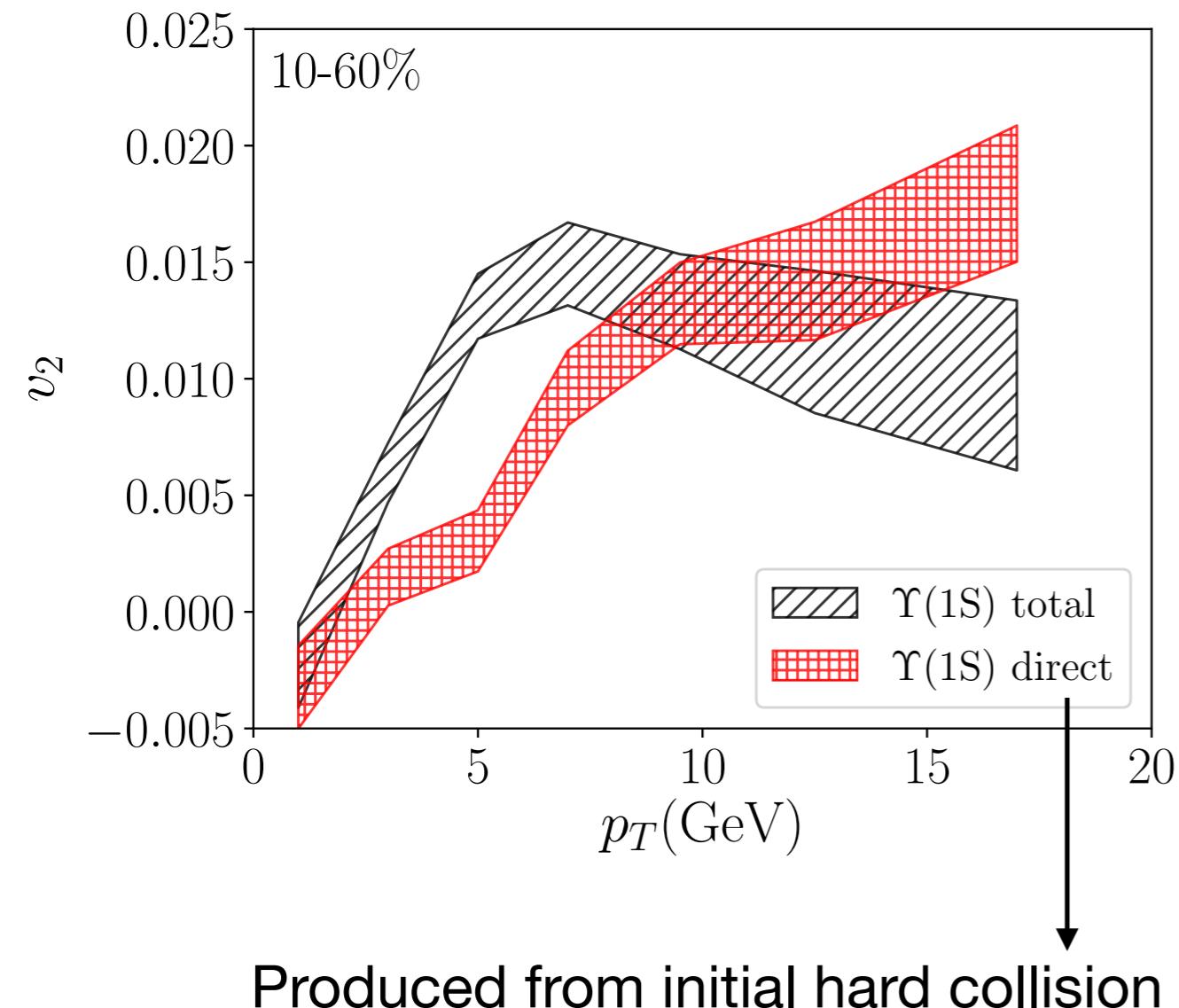
Use same set of parameters

sPHENIX kinematics $|y| < 1$



Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

$$E \frac{d^3 N}{dp^3} = \frac{1}{2\pi} \frac{d^2 N}{p_T dp_T dy} (1 + 2v_2 \cos(2\phi) + \dots)$$



(see talk by B.Paul (ALICE) at The 13th International Workshop on Heavy Quarkonium)

Conclusions

- Derivation of transport equations from open quantum system and effective field theory, importance of separation of scales
- Phenomenological results on bottomonium production in heavy ion collisions
- Future considerations:
 - Nonperturbative matching for pNRQCD using Wilson loop on lattice?

A. Rothkopf, T. Hatsuda and S. Sasaki, arXiv:1108.1579



Backup: Weak Coupling Expansion

$$H = H_S + H_E + H_I \quad H_I = \sum_{\alpha} O_{\alpha}^{(S)} \otimes O_{\alpha}^{(E)}$$

Redefine H_S so that $\langle O_{\alpha}^{(E)} \rangle \equiv \text{Tr}_E(O_{\alpha}^{(E)} \rho_E) = 0$

Expand unitary evolution to second order: $\theta(t) = (1 + \text{sign}(t))/2$

$$\begin{aligned} \rho_S(t) = \text{Tr}_E(\rho(t)) &= \rho_S(0) + \sum_{\alpha, \beta} \int_0^t dt_1 \int_0^t dt_2 C_{\alpha\beta}(t_1, t_2) \left(O_{\beta}^{(S)}(t_2) \rho_S(0) O_{\alpha}^{(S)}(t_1) \right. \\ &\quad \left. - \theta(t_1 - t_2) O_{\alpha}^{(S)}(t_1) O_{\beta}^{(S)}(t_2) \rho_S(0) - \theta(t_2 - t_1) \rho_S(0) O_{\alpha}^{(S)}(t_1) O_{\beta}^{(S)}(t_2) \right) + \mathcal{O}(H_I^3) \end{aligned}$$

Correlators of environment operators

$$C_{\alpha\beta}(t_1, t_2) \equiv \text{Tr}_E(O_{\alpha}^{(E)}(t_1) O_{\beta}^{(E)}(t_2) \rho_E)$$

Insert complete sets of system, define Lindblad operators

$$L_{ab} \equiv |a\rangle\langle b| \quad |a\rangle \text{ Eigenstates of } H_S$$

Backup: Basis Change in Relative Motion

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r \text{Tr} \left(S^\dagger (i\partial_0 - H_s) S + O^\dagger (iD_0 - H_o) O + V_A (O^\dagger \mathbf{r} \cdot g\mathbf{E} S + \text{h.c.}) + \frac{V_B}{2} O^\dagger \{ \mathbf{r} \cdot g\mathbf{E}, O \} + \dots \right)$$

Bra-ket notation in relative motion

$$S(\mathbf{R}, \mathbf{r}, t) = \frac{1}{\sqrt{N_c}} S(\mathbf{R}, \mathbf{r}, t) \equiv \frac{1}{\sqrt{N_c}} \langle \mathbf{r} | S(\mathbf{R}, t) \rangle$$

$$O(\mathbf{R}, \mathbf{r}, t) = \frac{1}{\sqrt{T_F}} O^a(\mathbf{R}, \mathbf{r}, t) T^a \equiv \frac{1}{\sqrt{T_F}} \langle \mathbf{r} | O^a(\mathbf{R}, t) \rangle T^a$$

Dipole interaction $\sqrt{\frac{T_F}{N_C}} \left(\langle O^a(\mathbf{R}, t) | \mathbf{r} \cdot g\mathbf{E}^a(\mathbf{R}, t) | S(\mathbf{R}, t) \rangle + \text{h.c.} \right)$



Composite fields

$$|S(\mathbf{R}, t)\rangle = \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} e^{-i(Et - \mathbf{p}_{\text{cm}} \cdot \mathbf{R})} \left(\sum_{nl} a_{nl}(\mathbf{p}_{\text{cm}}) \otimes |\psi_{nl}\rangle + \int \frac{d^3 p_{\text{rel}}}{(2\pi)^3} b_{\mathbf{p}_{\text{rel}}}(\mathbf{p}_{\text{cm}}) \otimes |\psi_{\mathbf{p}_{\text{rel}}}\rangle \right)$$

$$|O^a(\mathbf{R}, t)\rangle = \int \frac{d^3 p_{\text{cm}}}{(2\pi)^3} e^{-i(Et - \mathbf{p}_{\text{cm}} \cdot \mathbf{R})} \int \frac{d^3 p_{\text{rel}}}{(2\pi)^3} c_{\mathbf{p}_{\text{rel}}}^a(\mathbf{p}_{\text{cm}}) \otimes |\psi_{\mathbf{p}_{\text{rel}}}\rangle \quad E = -|E_{nl}|, \quad \frac{p_{\text{rel}}^2}{M} \quad \text{Power counting}$$

Quantization $[a_{n_1 l_1}(\mathbf{p}_{\text{cm}1}), a_{n_2 l_2}^\dagger(\mathbf{p}_{\text{cm}2})] = (2\pi)^3 \delta^3(\mathbf{p}_{\text{cm}1} - \mathbf{p}_{\text{cm}2}) \delta_{n_1 n_2} \delta_{l_1 l_2}$

$$[c_{\mathbf{p}_{\text{rel}1}}^{a_1}(\mathbf{p}_{\text{cm}1}), c_{\mathbf{p}_{\text{rel}2}}^{a_2\dagger}(\mathbf{p}_{\text{cm}2})] = (2\pi)^6 \delta^3(\mathbf{p}_{\text{cm}1} - \mathbf{p}_{\text{cm}2}) \delta^3(\mathbf{p}_{\text{rel}1} - \mathbf{p}_{\text{rel}2}) \delta^{a_1 a_2}$$

Backup: Correction of Potential

$$-i \sum_{ab} \sigma_{ab}(t) [L_{ab}, \rho_S(0)]$$

$$\begin{aligned} \sum_{a,b} \sigma_{ab} L_{ab} &\rightarrow t \sum_{n,l} \int \frac{d^3 k}{(2\pi)^3} \text{Re} \left\{ -ig^2 C_F \sum_{i_1, i_2} \int \frac{d^4 q}{(2\pi)^4} \int \frac{d^4 p_{\text{cm}}}{(2\pi)^4} \int \frac{d^3 p_{\text{rel}}}{(2\pi)^3} \right. \\ &\quad (2\pi)^4 \delta^3(\mathbf{k} - \mathbf{p}_{\text{cm}} - \mathbf{q}) \delta(E_k - p_{\text{cm}}^0 - q^0) \\ &\quad (q_0^2 \delta_{i_1 i_2} - q_{i_1} q_{i_2}) \left(\frac{i}{q_0^2 - \mathbf{q}^2 + i\epsilon} + n_B(|q_0|)(2\pi) \delta(q_0^2 - \mathbf{q}^2) \right) \\ &\quad \left. \langle \psi_{nl} | r_{i_1} \frac{i | \Psi_{\mathbf{p}_{\text{rel}}} \rangle \langle \Psi_{\mathbf{p}_{\text{rel}}} |}{p_{\text{cm}}^0 - E_p + i\epsilon} r_{i_2} | \psi_{nl} \rangle \right\} L_{|\mathbf{k}, nl, 1\rangle \langle \mathbf{k}, nl, 1|} \end{aligned}$$

